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Second year internship at IMFT in collaboration with Oregon State University

Study of groundwater in meadows alongside the Middle Fork John Day River, Oregon, USA

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Abstract

The restoration of the Middle Fork John Day River, Oregon, USA, involves modifications of the bed, the course and the banks of the river. In order to forecast the consequences of such modifications, it is important to understand that a river is just a part of a system with many interactions at stake. The purpose of this work is to study those interactions between the river, the water-table and the atmosphere. During summer data show that the water-table near the river drops with a relatively constant rate. We computed an average rate of evapotranspiration of 2.2mm/day using the White method. Data from Agrimet give a maximum possible evapotranspiration of 5mm/day. Yet, the water-table would drop too quickly if there were only evapotranspiration. With the White method we also found a constant groundwater inflow of 1.9mm/day. The difference between the evapotranspiration and the groundwater upwelling is the reason of the drop of the water-table during summer. A slower and not specific to the summer (as the evapotranspiration) phenomenon is the drainage of water from the soil to the stream. By using Darcy's Law we computed a flux of 0.002 to 0.02cfs added per kilometer of stream, with a time-constant of about 100 days. An estimation of water loss in soil profile confirmed the amount of water lost because of evapotranspiration. To study the shape of the water-table and check the value of the hydraulic conductivity we used an analytical approach with Boussinesq equation. Even though we could only partially confirm the theoretical shape of the water-table, we confirmed that a mean value for the hydraulic conductivity is 5m/day.

Résumé

La réstoration de la Middle Fork John Day River, Oregon, USA, implique des modifications du lit, du tracé et des berges de la rivière. Pour prévoir les conséquences de telles modifications, il est important de comprendre qu'une rivière n'est qu'un des éléments d'un système où de nombreuses intéractions sont en jeu. L'objectif de ce projet est l'étude de ces intéractions entre la rivière, la nappe phréatique et l'atmosphère. Les données mesurées sur le site au cours de l'été montrent que le niveau de la nappe phréatique diminue à un rythme constant .Nous avons calculé un taux moyen d'évapotranspiration de 2.2mm/jour en utilisant la méthode de White. D'après les données du site Agrimet l'évapotranspiration maximale possible est de 5mm/jour. Cependant, la nappe phréatique descend trop lentement pour que seule l'évapotranspiration soit en jeu. La méthode de White nous a permis de trouver un apport d'eau souterraine de 1.9mm/jour. C'est la différence entre cet apport d'eau souterraine et l'évapotranspiration qui explique la chute de la nappe phréatique.Un autre phénomène, plus lent et non spécifique à l'été comme l'évapotranspiration, est le phénomène de drainage de l'eau du sol vers la rivière. Grâce à la loi de Darcy nous avons calculé un flux de $6.5.10^{-5}$ à $6.5.10^{-4} m^3/s$ qui s'ajoutent par kilomètre de rivière, avec une constante de temps d'environ 100 jours. Une estimation de la perte d'eau dans le profil du sol a confirmé la quantité d'eau perdue par évapotranspiration. Pour étudier la forme de la nappe phréatique et vérifier la valeur de la conductivité hydraulique (ici équivalente à la perméabilité) nous avons utilisé une approche analytique avec l'équation de Boussinesq. Bien que n'ayant pu confirmer que partiellement la forme théorique de la nappe phréatique, nous avons confirmé la valeur moyenne de la perméabilité qui est de 5m/jour.

Introduction

I did a ten-week internship at IMFT (Institut de Mécanique des Fluides de Toulouse, France) on a project provided by the Oregon State University, Oregon, USA. My IMFT supervisor was M. Debenest, from the department of Porous media. The IMFT is a research laboratory which gathers about two hundred people and which is a leading European laboratory in Fluid Dynamics. The Porous Media Group works at developing knowledge in the field of transport in heterogeneous media such as fractured media, composites, biological media etc. The Biological and Ecological Engineering Department of Oregon State University has ongoing research in ecological engineering and water resource engineering with many active projects. The goal of this internship was to understand several phenomena that occur in the Middle Fork John Day River (MFJD River) in the northern Willamette Basin, Oregon. This river is currently under restoration and the Oregon State University provides a team that studies the river and gives advice about hydrology and related fields. The head of this team, professor Selker, was my supervisor. The restoration is made to implement US environmental laws and because the river crosses Native Americans' territories. There are several teams from governmental agencies and Oregon agencies that work on this restoration. As a result the river is well equipped and there are many sensors such as temperature sensors, discharge sensors, wells etc. I focused on the water-table height in the meadows along the river. The restoration of the river is aiming at having a sufficient water height in the river during the driest months and having plants, animals, and fishes both beside and into the river which explains our interest for the water table in the riverbanks. The variations of the water-table height within the soil on both sides of the river are mostly due to three phenomena: the drainage of the groundwater into the river, the evapotranspiration (ET) of the water through the plants and the input of groundwater coming from hills and mountains around the river. These are the three transfers I studied to have an idea of the amounts of water at stake.

In the first part of the report I give a description of the problem and I present general data. Then, in the second part, I identify the water loss and gain processes. In particular, I estimate the lateral flux of water drained into the river, the evapotranspiration, the change in soil water storage and the groundwater upwelling. Finally, in the last part, I present a more analytical approach to the problem of bank drainage. The focus is on the shape of the water-table. I compute solutions to the Boussinesq equation and I compare it to the data.

I-Understanding the problem

1-Description

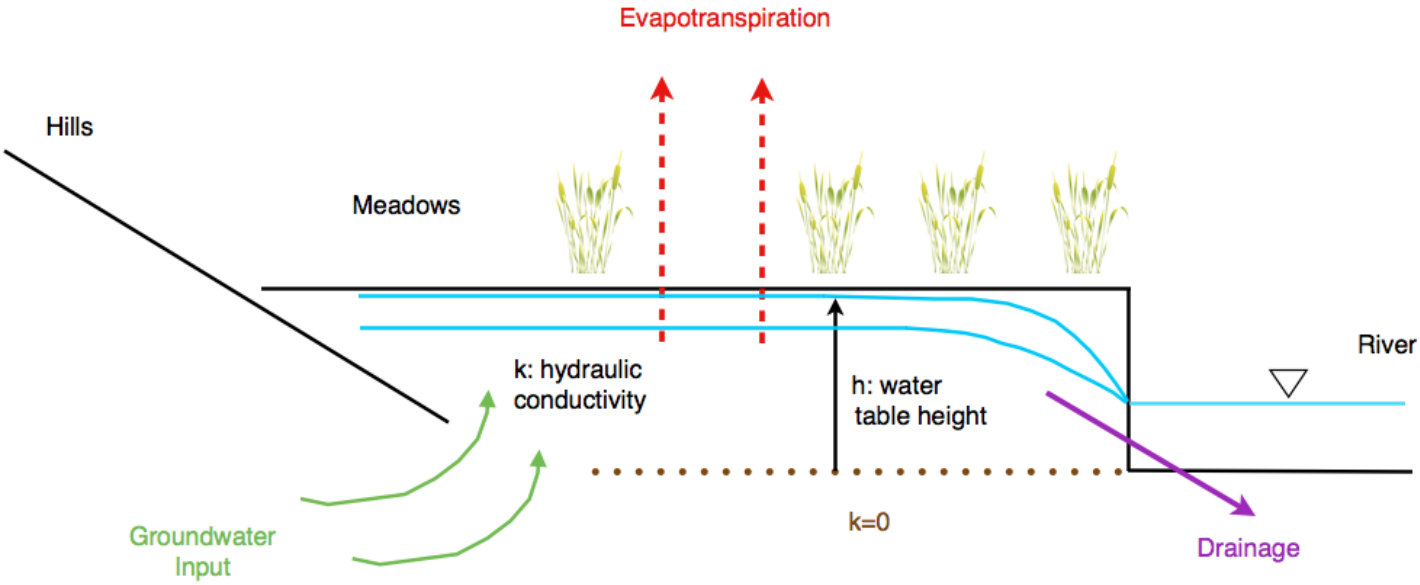

Figure 1: Description of the simplified problem showing a sketch of an early water-table and the water-table at some time later when it has fallen due to drainage to the stream and ET

As we can see on the figure 1, I considered a simplified symmetrical model of the river, with meadows on both sides. The river is taken to be about one meter deep and four meters wide, and the meadows are about a hundred meters in width. It originates in the Blue Mountains of the Malheur National Forrest and flows westerly for 75 miles (120 km). The watershed drains an area of 806 mi^2 (2087 km^2) and receives an approximately 15-25 inches of precipitation each year (380-630 mm). The stream flow is 255 cfs (cubic feet per seconds) on average ($7m^3/s$) (all data from the Experimental Design and Implementation Plan, Upper Middle Fork John Day Group [1]). The variations of the water-table height are due to the drainage of the water into the river, the evapotranspiration through the plants (mostly grass and a few trees) and the upwelling of groundwater coming from upslope areas. During a normal year the water level in the river and the water-table height in meadows vary of several tens of centimeters. These variations can be separated into two signals. First a yearly signal where we can see the elevation of the water-table due to precipitation and the absence of evapotranspiration during fall and winter, then during the end of winter and spring a stabilization of the water-table, and finally the decrease of the water-table due to evapotranspiration and the lack of precipitation during summer. Secondly a daily signal which mostly appears during summer when the water-table decreases and that we used to estimate the groundwater upwelling and the evapotranspiration employing the White method (see the two publications: L.K.Lautz (2007) [2] and S.P.Loheide, J.J.Butler and S.M.Gorelick (2005) [3]).

2-General data

I was provided data from the site by Tara O'Donnell, an MS Student working with Dr. Selker. Here I present some general data to help to understand the physical processes which predominate in our system. Most of the data come from the wells presented on figure 4 and figure 5.

Figure 2: Situation of the river

Figure 3: The Middle Fork John Day River

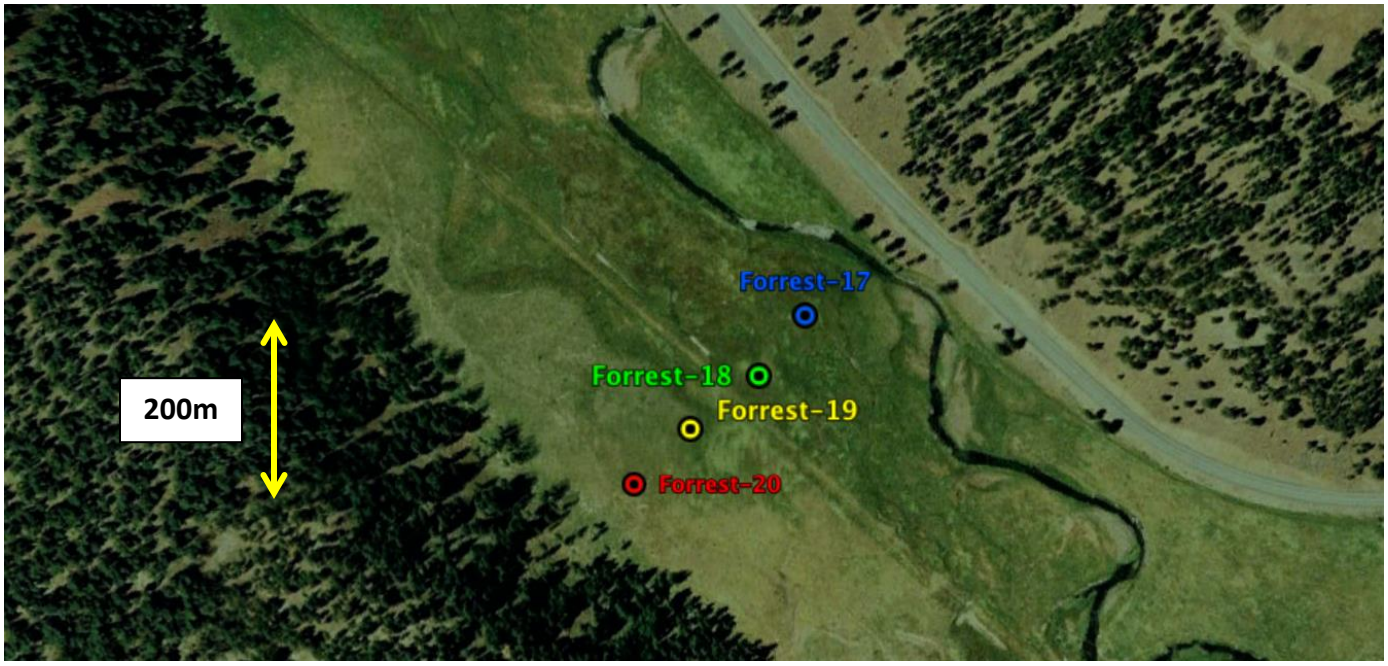

Figure 4: The four wells situated near the river at Forrest

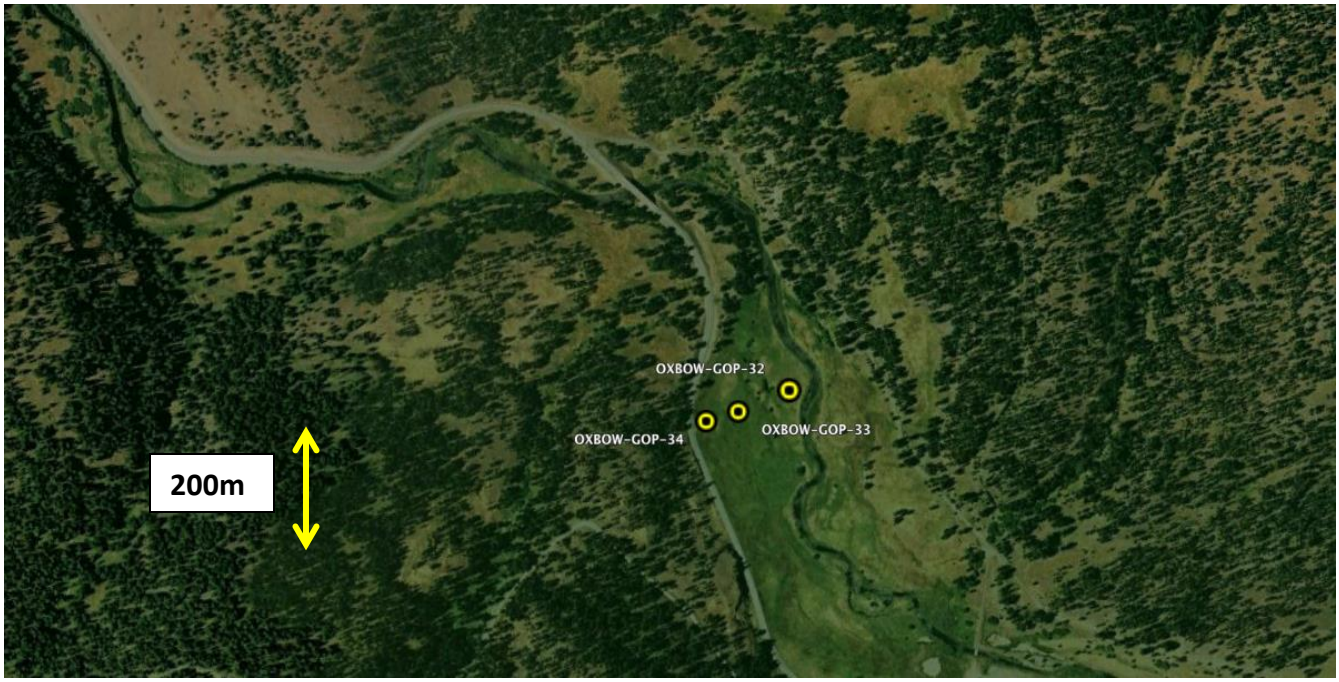

Figure 5: The three wells situated near the river at Oxbow

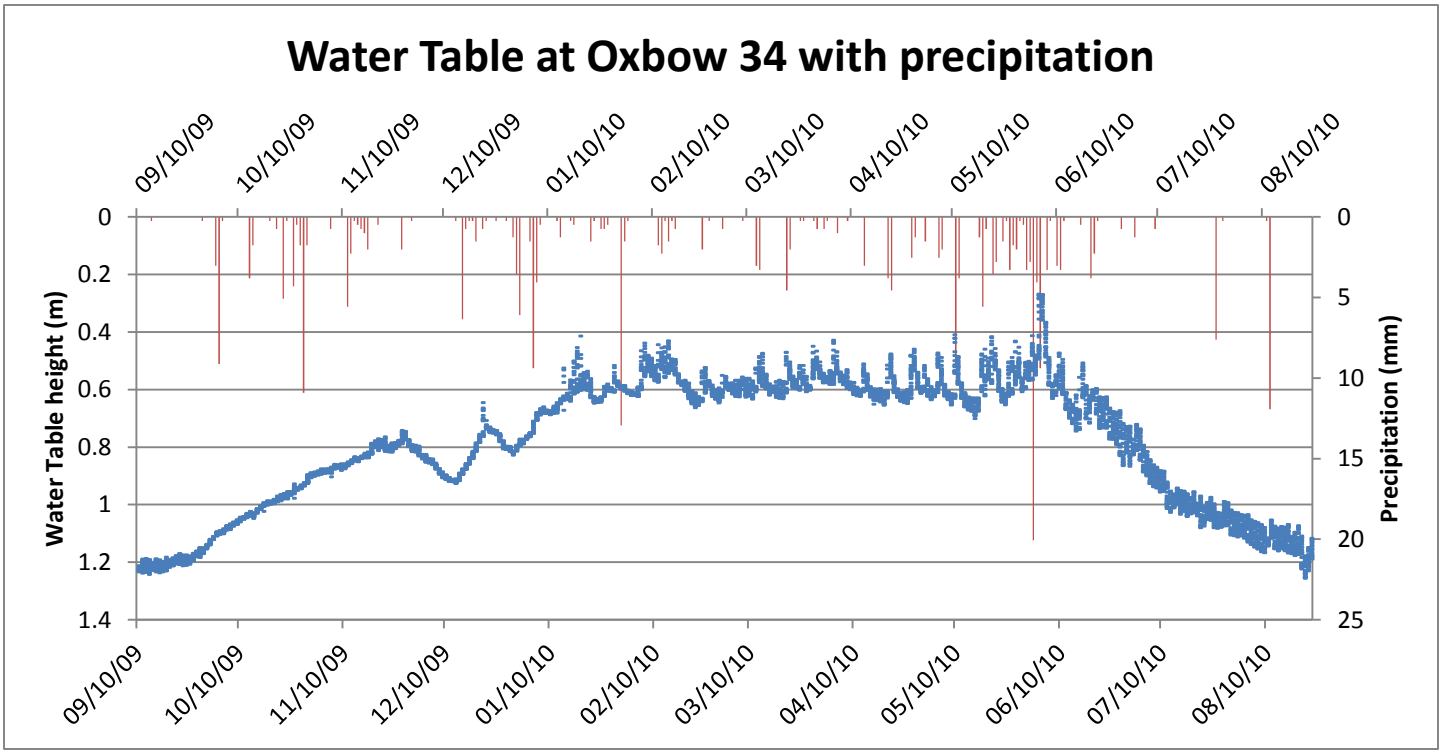

Figure 6: Evolution of the water table during the 2010 water-year

The figure 6 shows the evolution of the water-table during a “normal” year. Those data have been recorded during 2009 and 2010, as most of the data presented in this report. The red lines represent the precipitation that I downloaded from the Agrimet website (Agrimet [4]). Those precipitations were measured at Prairie City station, 25 km southerly to the location of the wells (a distance which may induce some discrepancies between precipitation and water table variations). Anyway, we can see that the water-table height does not increase only when a rainfall occurs. The reason is that the main provider of water is the snowmelt. However, major rainfalls can be easily related to elevations of the water-table on this figure. This figure also illustrates the yearly signal described above with the three seasonal steps: increase, stabilization, and decrease of the water-table height. A blow-up of the last part of the curve highlights the daily signal (figure 7).

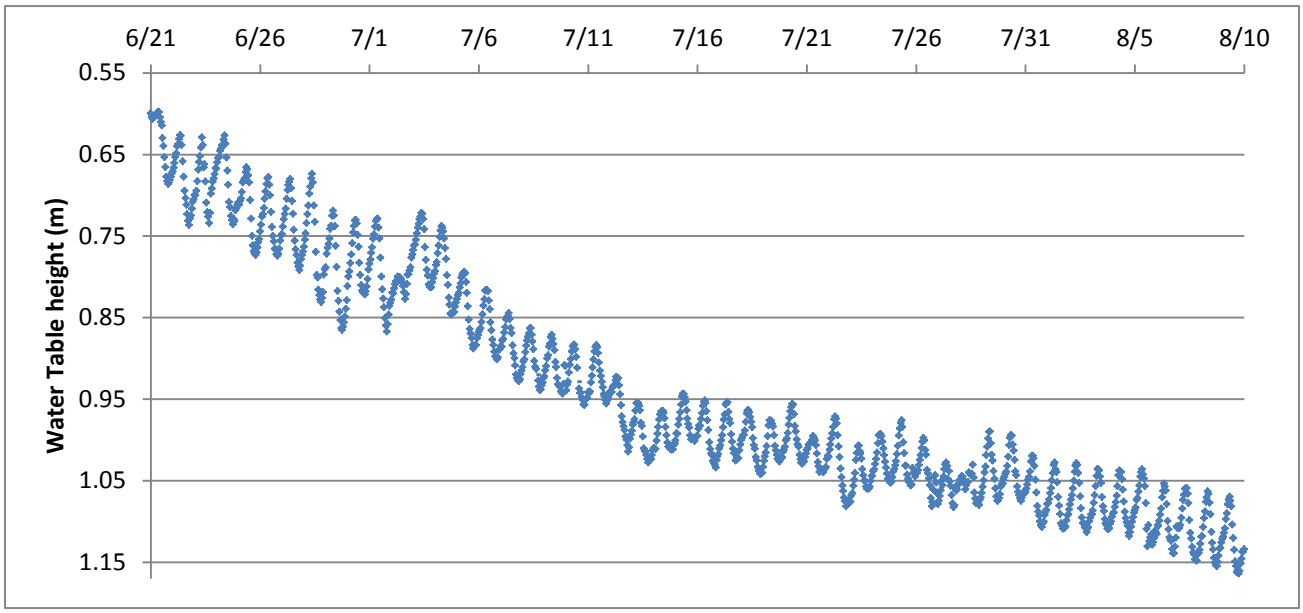

Figure 7: Daily signal during the decrease of the water-table height at Oxbow 34 during summer 2010

The data in figure 7 show clearly the daily water table drops due to intense evapotranspiration. We will study this signal in detail later, but what is interesting to note now is that the water table is always at its highest around 7:00 and at its lowest around 17:00. These daily variations will allow us to compute the groundwater recharge rate and the evapotranspiration by using the White Method.

The estimates of evapotranspiration nearby Prairie City can be obtained from the Agrimet website [4] (figure 8). These are the data I used to check if the values of the evapotranspiration I computed were reasonable. What we can see is that the evapotranspiration start to be appreciable in the spring (April), increases until July, and then decreases until the end of September when it drops to less than half of its peak value. The specific values for each crop are obtained with crop coefficients from the reference evapotranspiration.

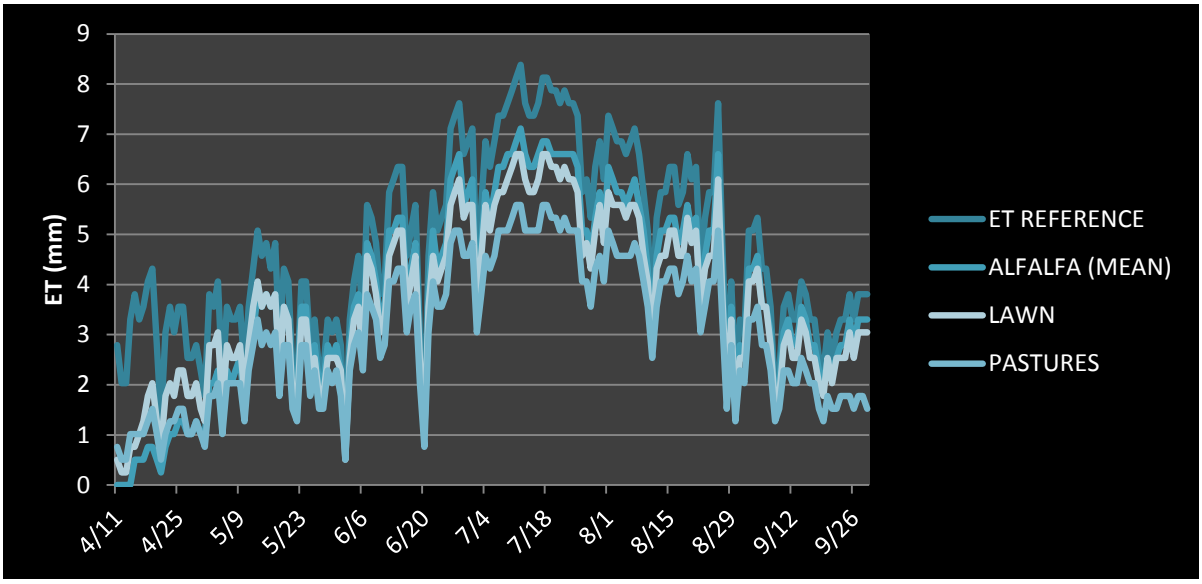

Figure 8: Evolution of the rate of evapotranspiration at Prairie City during the summer of 2010

II-Identification of water loss and gain processes

1-Estimation of lateral flux

In order to compute the lateral flux of groundwater that goes into the river we use Darcy's Law. This law can be written as the following equation:

$$u = -k_s \frac{\partial h}{\partial x} \; (Darcy's law) \qquad (\boldsymbol{Eq\ 1})$$

where u (m/day) is the groundwater flux, k_s (m/day) is the saturated hydraulic conductivity (also called permeability), h (m) is the water-table height and x (m) is the abscise perpendicularly to the river. If we multiply both terms by the area of drainage A (m^2), we obtain a relationship between the flux of groundwater and the gradient: $Q = A * u \; (m^3/day)$.

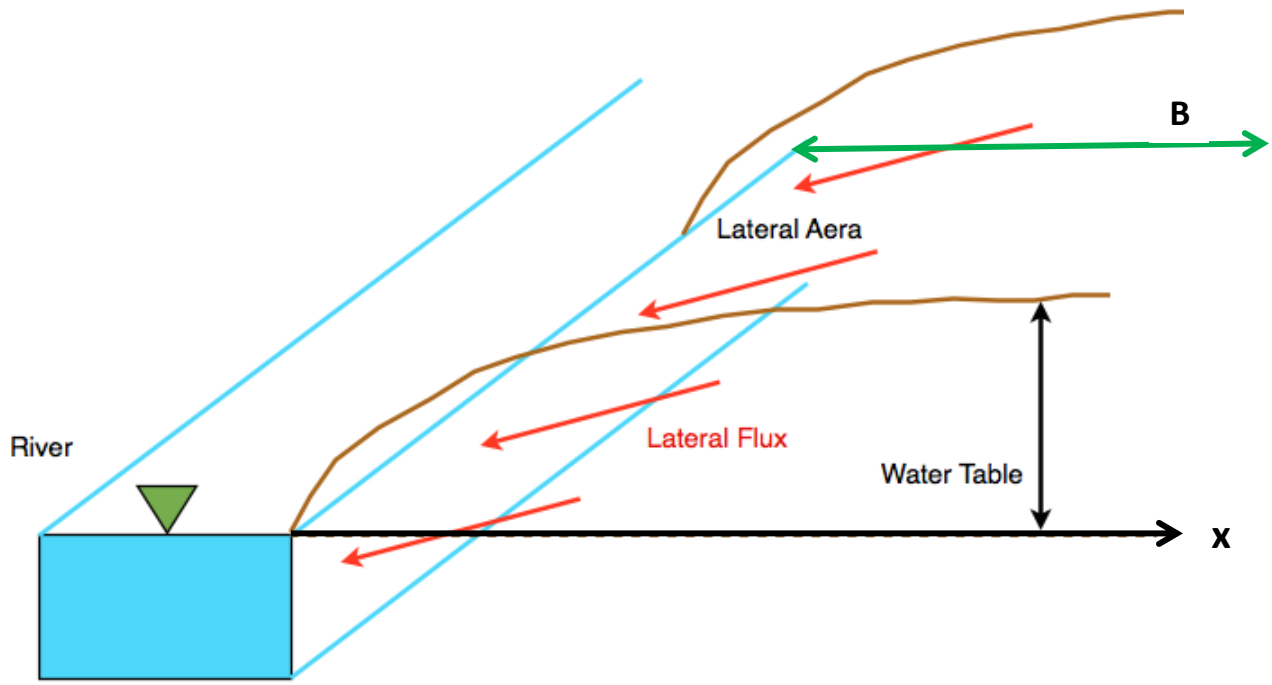

Figure 9: Sketch of the problem

According to two different soil surveys, one for the whole Grant County (the county where our river is to be found), and one done in the banks of the river, the saturated hydraulic conductivity has an average value of approximately 5 m/day, but can range from 0.02m/day to 20m/day depending on the depth and the soil composition.

I computed different values of the lateral flux for the different values of the hydraulic conductivity and for different values of the water-table gradient (which varies during a year). I chose a drainage area of 1.5m in depth by 1km in length in order to obtain the flux in m^3 per kilometer of river per day. I assumed that B is the distance from the river to the last well (the farthest to the river), i.e. 200m at Oxbow and 240m at Forrest. I also computed a characteristic time constant τ that represents the time at which the drainage dropped to half of its initial value. I used the solution of the linearized Boussinesq equation that is presented in part III-2-c of this report.

$$h(x,\tau) = 0.5h(x,t=0) \qquad (\boldsymbol{Eq\ 2})$$

So when replacing h by its expression the equation becomes

$$Dsin\left(\frac{\pi x}{2B}\right)\exp\left(-\frac{kh_0\pi^2}{4\varphi B^2}\tau\right) = 0.5Dsin\left(\frac{\pi x}{2B}\right) \qquad (\boldsymbol{Eq\ 3})$$

$$\xrightarrow{yields} \exp\left(-\frac{kh_0\pi^2}{4\varphi B^2}\tau\right) = 0.5$$

$$\xrightarrow{yields} -\frac{kh_0\pi^2}{4\varphi B^2}\tau = \ln(0.5) = -\ln(2)$$

Finally, the expression of τ is:

$$\tau = \frac{4\ln(2)\varphi B^2}{kh_0\pi^2}\ (days)$$

where φ is the specific yield, B the width of the meadow (approximately equal to dx), h_0 the average water-table height and k the hydraulic conductivity.

All the results are presented in the tables below.

Forrest				Q (m3/day/km)				
Depth (m)				k (m/day)	0.01	0.1	1	10
1.5	dh (m)	dx (m)	dh/dx					
Length (m)	0.5	180	0.0028		0.042	0.42	4.2	42
1000	0.6		0.0033		0.05	0.5	5	50
Area (m2)	0.7		0.0039		0.058	0.58	5.8	58
1500	0.8		0.0044		0.067	0.67	6.7	67
B (m)	0.9		0.005		0.075	0.75	7.5	75
180	1		0.0056		0.083	0.83	8.3	83
Specific yield	1.1		0.0061		0.092	0.92	9.2	92
0.04	1.2		0.0067		0.1	1	10	100
ho (m)								
1.5				t 1/2 (days)	10541	1054	105	10.5

Table 1: Lateral flux at Forrest

Oxbow				Q (m3/day/km)				
Depth (m)				k (m/day)	0.01	0.1	1	10
1.5	dh (m)	dx (m)	dh/dx					
Length (m)	0.4	160	0.0025		0.038	0.38	3.75	38
1000	0.5		0.0031		0.047	0.47	4.67	47
Area (m2)	0.6		0.0038		0.056	0.56	5.6	56
1500	0.7		0.0048		0.066	0.66	6.6	66
B (m)	0.8		0.005		0.075	0.75	7.5	75
160	0.9		0.0056		0.084	0.84	8.4	84
Specific yield								
0.04				t 1/2 (days)	8329	833	83	8.3
ho (m)								
1.5								

Table 2: Lateral flux at Oxbow

So now what can we infer from these results? Clearly, the term with the greatest impact on the flux is the hydraulic conductivity with a factor of 10^4 between the upper and the lower values. The water table gradient (which, as we said before, changes during a year) has a less important impact on the flux with a factor of approximately 2 between the upper and the lower values.

Then it is interesting to notice that because of the strong influence of k on the lateral flux its values range from 0.03m^3/day/km (3.5 $10^{-8}m^3$/s/km or 1.2 10^{-5}cfs/km) to 100m^3/day/km (0.0012m^3/s/km or 0.041cfs/km). As a result one may argue that we cannot draw any conclusion from so different results. However, since we are just doing estimations the most relevant values are the red ones as the mean value of k is between 1m/day and 10 m/day and dh is nearly always equal to 0.5m at Forrest and 0.7m at Oxbow. Thus we can conclude that Q approximately ranges from 5m^3/km/day (5.8 10^{-5} m^3/s/km or 0.0020cfs/km) to 50m^3/km/day (5.8 10^{-4} m^3/s/km or 0.020cfs/km) along the river. So the drainage is a quite slow phenomenon which does not involve a lot of water each day. This is supported by the characteristic time constant which is between 8 and 100 days. Indeed it means that it takes up to 100 days to drain half of the water stored in the ground. This is also significant in the potential utility of bank storage to augment late season flows. As the snowmelt last 90 days, it means that we still have an additional flow of approximately 50% of Q (0.0010cfs to 0.010cfs per kilometer).

2-Description of potential evapotranspiration (PET)

The potential evapotranspiration (PET) is the maximum evapotranspiration (ET) that would occur if a sufficient water source were available. It is obtained by meteorological measurements and computations. To have an idea of the PET around the MFJD River, I used data from Agrimet (The Pacific Northwest Cooperative Agricultural Weather Network). I downloaded data recorded at Prairie City Station [4], situated 25km southerly to the MFJD River (Figure 10).

Figure 10: Prairie City Agrimet Station [4]

The figure 11 presents a general view of the daily PET from spring 2009 to fall 2010 where we can see that the ET occurs mainly during spring and summer. The highest value reached is 8mm but during summer the PET is about 5mm on average. Figures 12 and 8 allow us to have a more precise idea of the values of the daily PET.

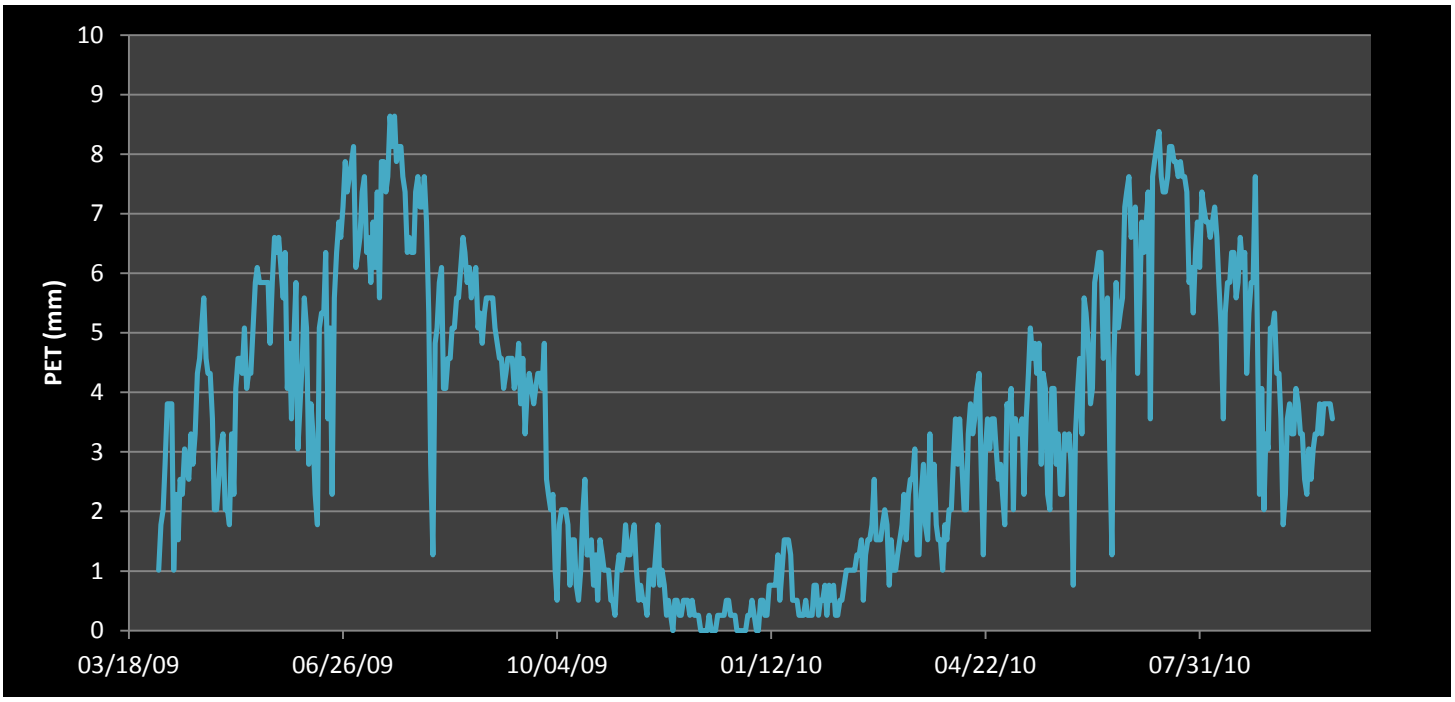

Figure 11: PET at Prairie City from spring 2009 to fall 2010

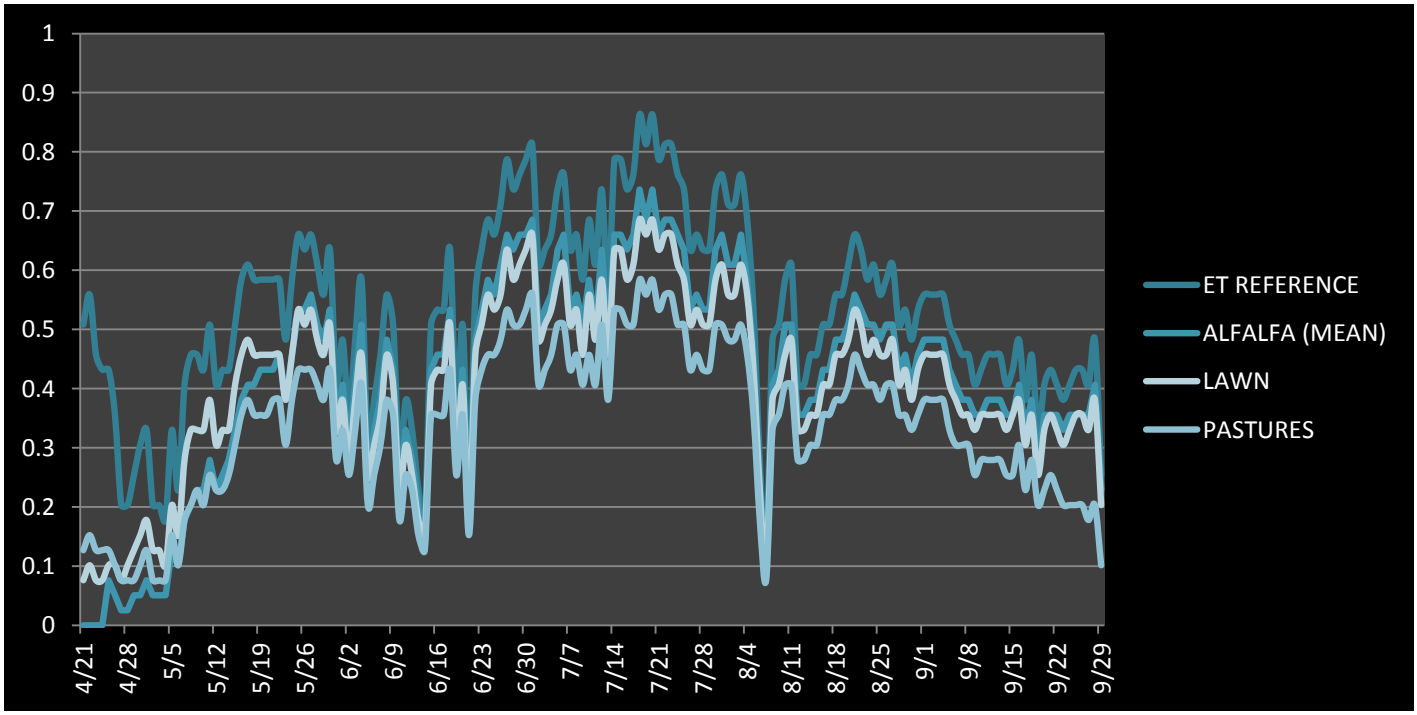

Figure 12: PET at Prairie City during summer 2009

Figures 12 and 8 show the computed ET for three crops: alfalfa, lawn and pastures. ET Reference represents PET and the others ET rates have been computed with crop coefficients by Agrimet. On both years we have similar variations: an increase of ET until mid-July, and then a decrease. The average values of daily ET are summarized in table 3.

Year	ET Reference (mm/day)	Alfalfa Mean (mm/day)	Lawn (mm/day)	Pastures (mm/day)
2009	5.3	4.1	4.0	3.3
2010	4.7	3.7	3.6	3.0

Table 3: Mean annual daily PET for different crops in 2009 and 2010

First, we can notice that ET was higher in 2009 than in 2010. The difference is of approximately 0.5mm per day which is considerable over a year. Then we see that depending on the year and the crop the daily ET ranges from 3 to 5mm. Those values are the reference values to which we will compare our results in the next part. We should keep in mind that those values are an "upper bound" of ET, and that we expect the real ET to be less important.

3-The White Method: estimation of ET and groundwater inflow

a) Presentation of the method

The White method consists in using the daily fluctuations of the water-table during summer, when ET causes the water-table to decrease, to estimate the groundwater inflow and the ET. In fact, what it allows us to compute is rather called a groundwater evapotranspiration (ET_G) and is only a part of the actual ET. As we are mostly interested in groundwater, this is well suited to our needs. Using White method to compute ET is an interesting method as it is direct and both calculations and field measurements are easy to do. However it is not a widespread method among the scientific community and only few publications show the link between ET and ET_G (see White (1932) [5] and [2] and [3]).

The White method uses the following equation to estimate ET_G:

$$ET_G = S_y(24r_{gw} \pm s) \qquad (\boldsymbol{Eq\ 4})$$

where S_y is the specific yield, r_{gw} the rate of water-table rise during the night and s the net rise or fall of water-table during a 24-h period. Therefore, to find out the amount of water consumed by the ET_G, we take the difference between the actual water-table height at the end of a day and the water-table elevation that we would have if there was only a groundwater flux (Figure 14).

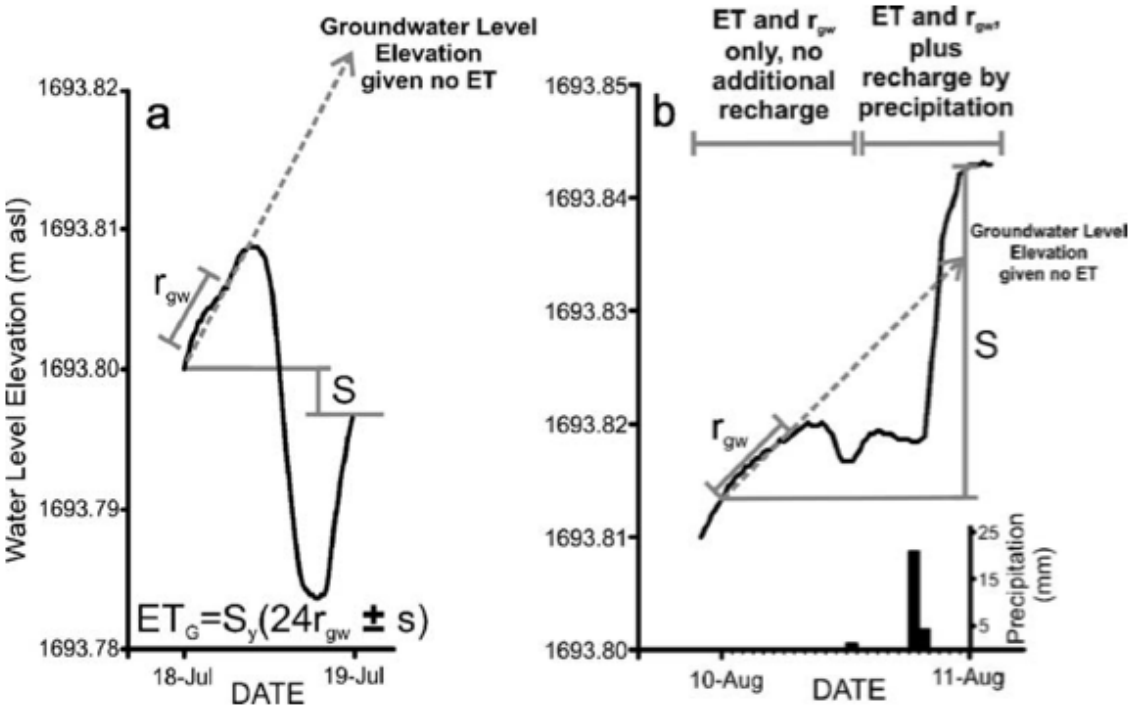

Figure 14: Diagrammatic explanation of the White method (from Lautz (2007) [2])

b) Sources of errors

To confirm the utility of the White method for our case, we listed the sources of errors to see whether the errors were large compared to the estimated flux.

The first thing to check is the assumption that daily fluctuations of water-table elevation are mainly due to ET. The two other causes of those fluctuations are variations of temperature (causing potential errors in pressure sensor readings) in the soil and variations of barometric pressure. In shallow systems, changes in barometric pressures are scarcely significant relative to ET_G. To be sure of that we can look at figure 15.

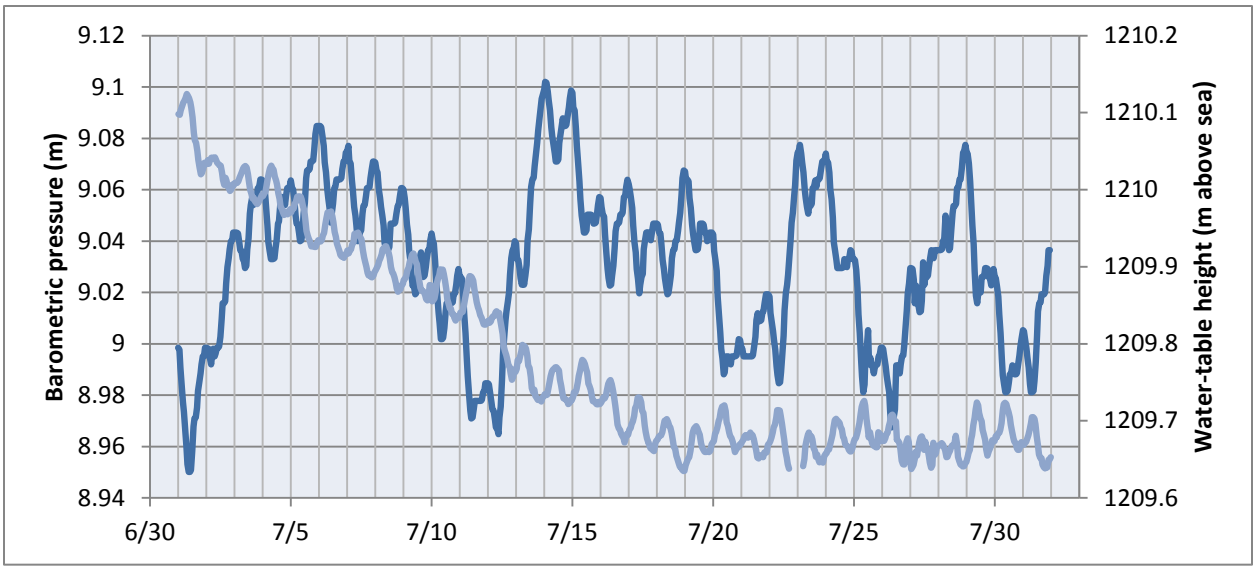

Figure 15: Barometric pressure and water-table at Forrest 20 during July 2010

It is clear on figure 15 that we do have daily fluctuations of barometric pressure (dark-blue line), mixed with fluctuations occurring in longer periods. However those daily fluctuations have amplitudes of about 0.5 to 2cm, which is not sufficient enough to have an effect on the water-table height situated one meter deep in the soil (which variations have amplitudes of 5cm). Besides, if those daily fluctuations had a visible impact on water-table height, the greater fluctuations that occur on longer periods would have an even greater impact on water-table height. Though we cannot see that supposed impact on the water-table height at Forrest 20 (light-blue line).

As I said before, it may also be the fluctuations of the water temperature that would cause our daily fluctuations. I plotted the temperatures recorded in the well at Forrest 20 during July 2010 on figure 16. Even if the curve's shape seems a bit odd, we can see that there are no daily fluctuations. Moreover, the temperature has an overall variation of less than 1.5°C for July 2010, so this is seen to be a really small variation, as one would expect from a sensor situated well below the soil surface.

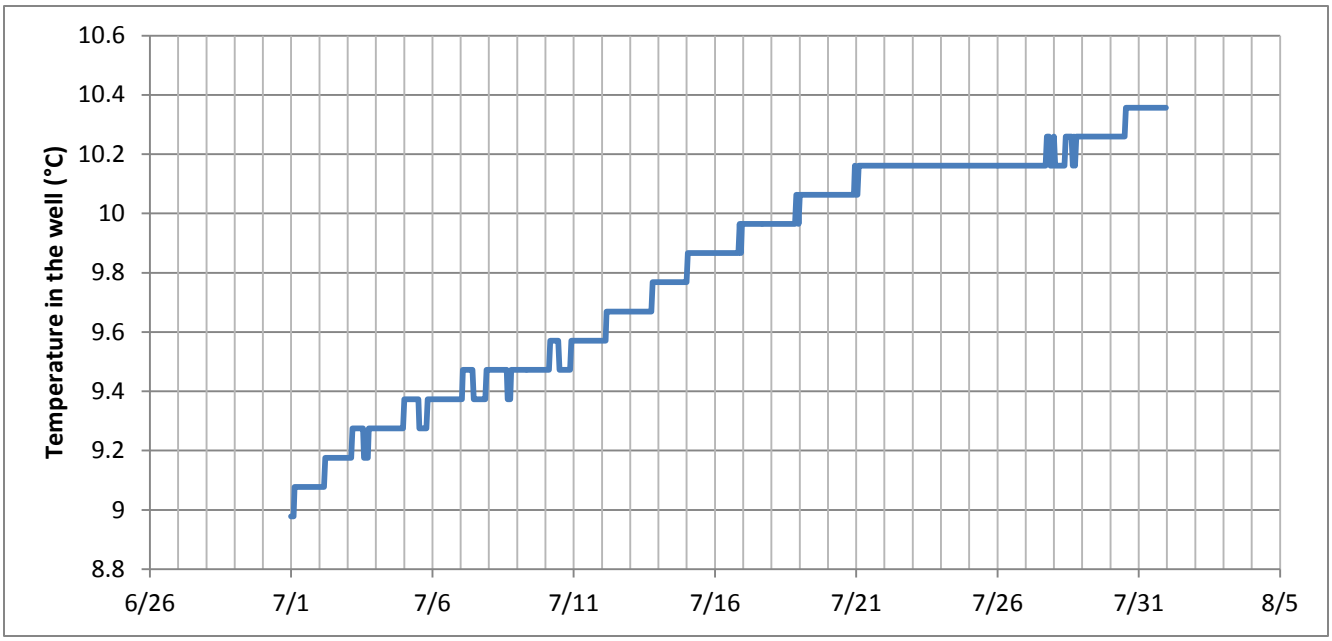

Figure 16: Temperature recorded in the well Forrest 20 in July 2010

The second source of error is in the parametric value of the specific yield S_y. Indeed, the White method's formula shows that ET_G is directly proportional to S_y. As a result, errors in the value of S_y result in proportional errors in the estimation of ET_G. In fact, we have to use the "readily available specific yield" which reflects the amount of water that is released from the saturated zone, per unit drop in the water table per unit land surface area, in the time frame of the diurnal fluctuations. The readily available specific yield is generally about half of the specific yield. We have to check its value in table 4 by using the composition of the soil. We know this composition thanks to the National Cooperative Soil Survey from the US Department of Agriculture [6]. It appears that we should use a value of about 0.02 for S_y, so a value of 0.04 for the normal specific yield. This value does not contradict the values presented in another soil survey for the whole Grant County [7].

									S_Y			
Sediment Texture	θ_S	θ_R	α	n	$K_{S,\ m/d}$	$S_{S,\ 1/m}$	Sand, %	Clay, %	$\theta_S - \theta_R$	Depth Compensated	From *Johnson* [1967]	Readily Available
Sand	0.43	0.045	14.5	2.68	7.1	0.0002	92.7	2.9	0.385	0.38	0.34	0.32
Loamy sand	0.41	0.057	12.4	2.28	3.5	0.0003	80.9	6.4	0.353	0.34	0.26	0.26
Sandy loam	0.41	0.065	7.5	1.89	1.1	0.0004	63.4	11.1	0.345	0.29	0.19	0.17
Loam	0.43	0.078	3.6	1.56	0.25	0.0005	40.0	19.7	0.352	0.19	0.095	0.075
Silt	0.46	0.034	1.6	1.37	0.060	0.0006	5.8	9.5	0.426	0.11	0.06	0.026
Silt loam	0.45	0.067	2.0	1.41	0.11	0.0006	16.6	18.5	0.383	0.12	0.07	0.037
Sandy clay loam	0.39	0.100	5.9	1.48	0.31	0.0008	54.3	27.4	0.290	0.17	0.05	0.072
Clay loam	0.41	0.095	1.9	1.31	0.062	0.0008	29.8	32.6	0.315	0.078	0.038	0.021
Silty clay loam	0.43	0.089	1.0	1.23	0.017	0.0007	7.6	33.2	0.341	0.041	0.029	0.012
Sandy clay	0.38	0.100	2.7	1.23	0.029	0.0010	47.5	41.0	0.280	0.068	0.025	0.015
Coarse sand	0.43	0.045	14.5	2.68	200	0.0002	-	-	0.385	0.38	-	0.38
Medium sand	0.43	0.045	14.5	2.68	50	0.0002	-	-	0.385	0.38	-	0.36
Fine sand	0.43	0.045	14.5	2.68	12.4	0.0002	-	-	0.385	0.38	-	0.33
Very fine sand	0.43	0.045	14.5	2.68	3.1	0.0002	-	-	0.385	0.38	-	0.31

Table 4: Specific yield for different soil compositions (from Loheide (2005) [3])

c) Presentation of the results

I wrote a Matlab script that processes automatically the data in order to compute the ET_G. With this script I computed ET_G and the groundwater inflow for the periods indicated in table 5. An example of those data is presented on figure 17.

	Forrest 17	Forrest 18	Forrest 19	Forrest 20	Oxbow 32	Oxbow 33	Oxbow 34	RPB 4	RPB 5	RPB6	RPB7
Starting date	07/01/10 00:00	07/01/10 00:00	07/01/10 00:00	07/01/10 00:00	06/21/10 00:00	06/21/10 00:00	06/21/10 00:00	06/21/10 00:00	06/21/10 00:00	06/21/10 00:00	06/21/10 00:00
Finishing date	07/31/10 23:00	07/31/10 23:00	07/31/10 23:00	07/31/10 23:00	08/21/10 23:00	08/20/10 23:00	08/21/10 23:00	08/21/10 23:00	08/21/10 23:00	08/21/10 23:00	08/21/10 23:00

Table 5: Data processed with Matlab

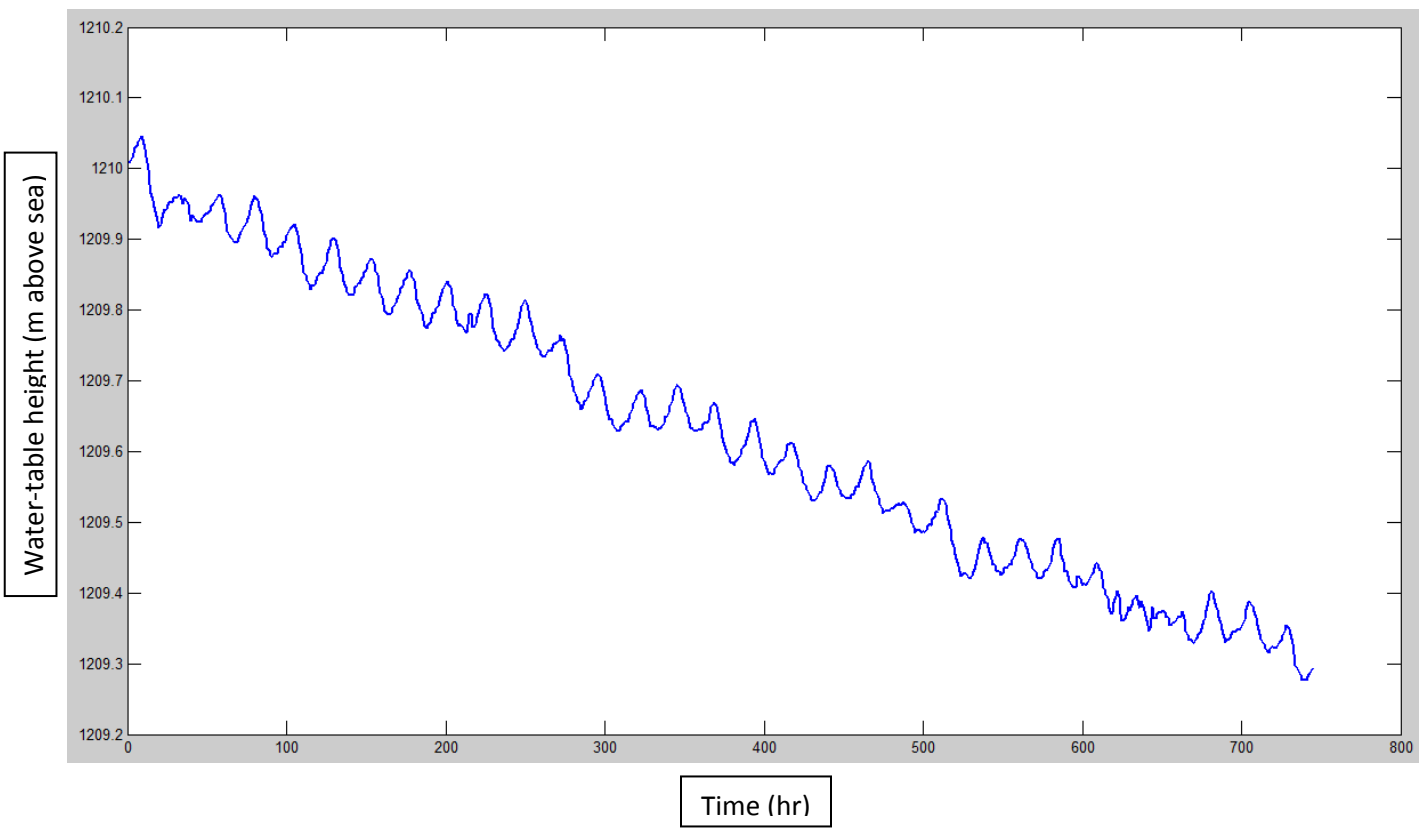

Figure 17: Daily fluctuations at Forrest 19 during July 2010

	Forrest 17	Forrest 18	Forrest 19	Forrest 20	Oxbow 32	Oxbow 33	Oxbow 34	RPB 4	RPB 5	RPB 6	RPB 7
ETg (mm/day)	2.4	2.7	2.2	1.8	2.1	2.6	2.7	1.9	2.0	1.9	2.1
Standard deviation(mm)	1.0	0.94	0.81	0.77	0.85	0.99	1.01	0.83	0.84	0.81	0.88
$24*r_{GW}$ (mm/day)	106	115	85	77	94	117	123	82	88	86	98
S_y*24*r_{GW} (mm/day)	2.1	2.3	1.7	1.5	1.9	2.3	2.5	1.6	1.8	1.7	2.0

Table 6: Mean daily values of ETg and the groundwater inflow

What we can see with table 6 is that the method gives quite consistent results. Indeed we find approximately the same amount of daily evapotranspiration for each well. Besides the standard deviation is rather high but acceptable given the fact that we are dealing with a phenomenon with a high variability in time. The average value of ET_G is 2.2mm. We will compare these values to the ones provided by Agrimet in next paragraph. For the groundwater upwelling we find an average of 1.9mm/day. One should be careful about the groundwater inflow: its value is equal to $Sy*24*r_{gw}$, and not directly $24*r_{gw}$. These results seem quite accurate as there are smaller than the Agrimet values but have the same order of magnitude. Moreover, we can see that ET_G is higher than the groundwater inflow, which is what we expected to find as the water-table decreases during the time-period we chose. During July 2010, the water-table drops of 72cm at Forrest 19 (see Figure 15 for visual confirmation). If we use our daily values of the evapotranspiration we find that the water-table drops of $(ET_G-S_y*24*r_{GW})*31/S_y$=77cm (cf Eq 1: the difference between the ET and the groundwater inflow multiplied by a month and divided by the specific yield). Thus, the method seems to be accurate.

The three following figures show scatter plots of ET_G versus PET provided by Agrimet. It is not easy to interpret those figures. Although we have a trend line that I calculated by taking the average of ET_G/PET for each well, there is substantial scatter around the trend line. In fact there is a standard deviation between points and trend line of 1.27mm for Forrest 17, 0.83mm for Oxbow 32 and 0.8mm for RPB4. It is likely to be due to environmental factors which are not taken into account in the computation of PET. However, we can notice that all the ET_G/PET ratios are around 0.3 which means that our results are rather consistent from a well to another. Indeed the standard deviation of the ratios of all the wells is only of 0.048.

Yet, we do not have enough data to conclude that the trend lines are absolutely correct. If we had had more wells, or data for several years, our conclusions would have been more reliable. What we can state confidently is that the amounts of water at stake in the ET_G do not contradict any other result. On the contrary it explains the drop of the water-table accurately and as expected the ET_G is smaller than the PET. By estimating the water loss in soil profile we will try to have another confirmation.

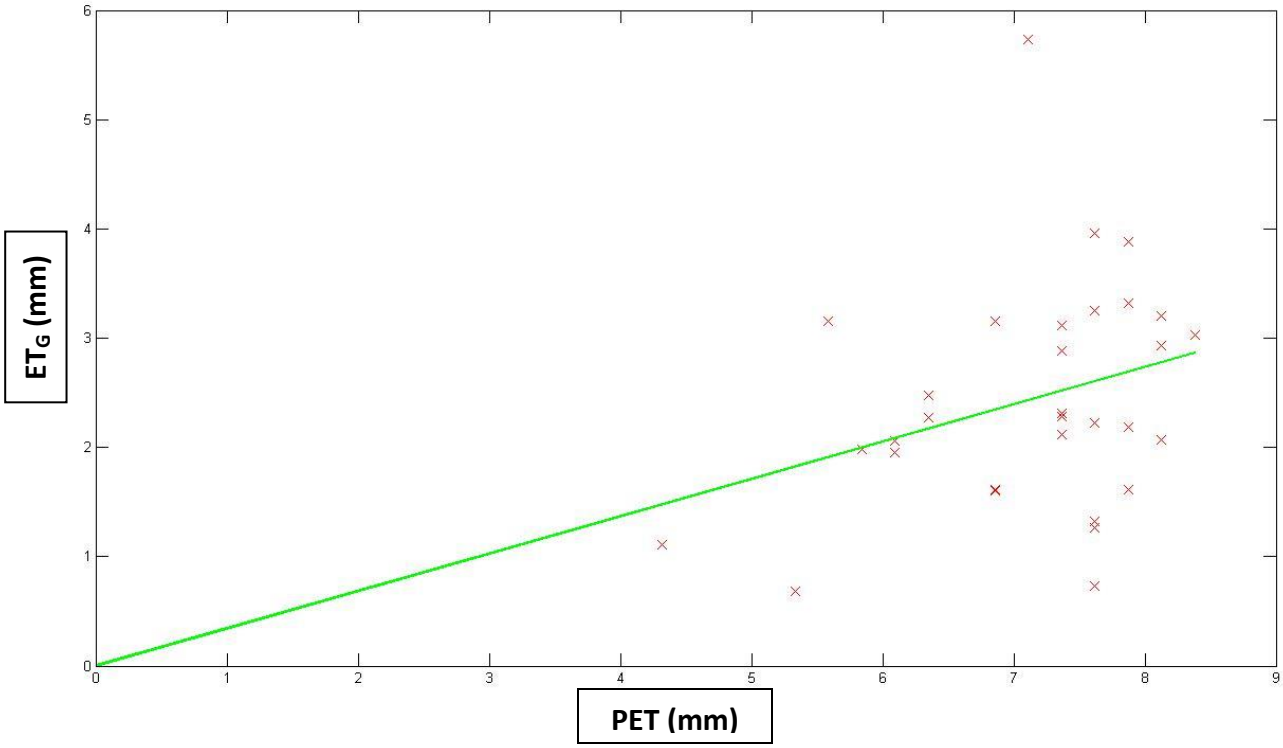

Figure 18: Scatter plot of ET_G against PET with a trend line with a coefficient of 0.34, Forrest 17

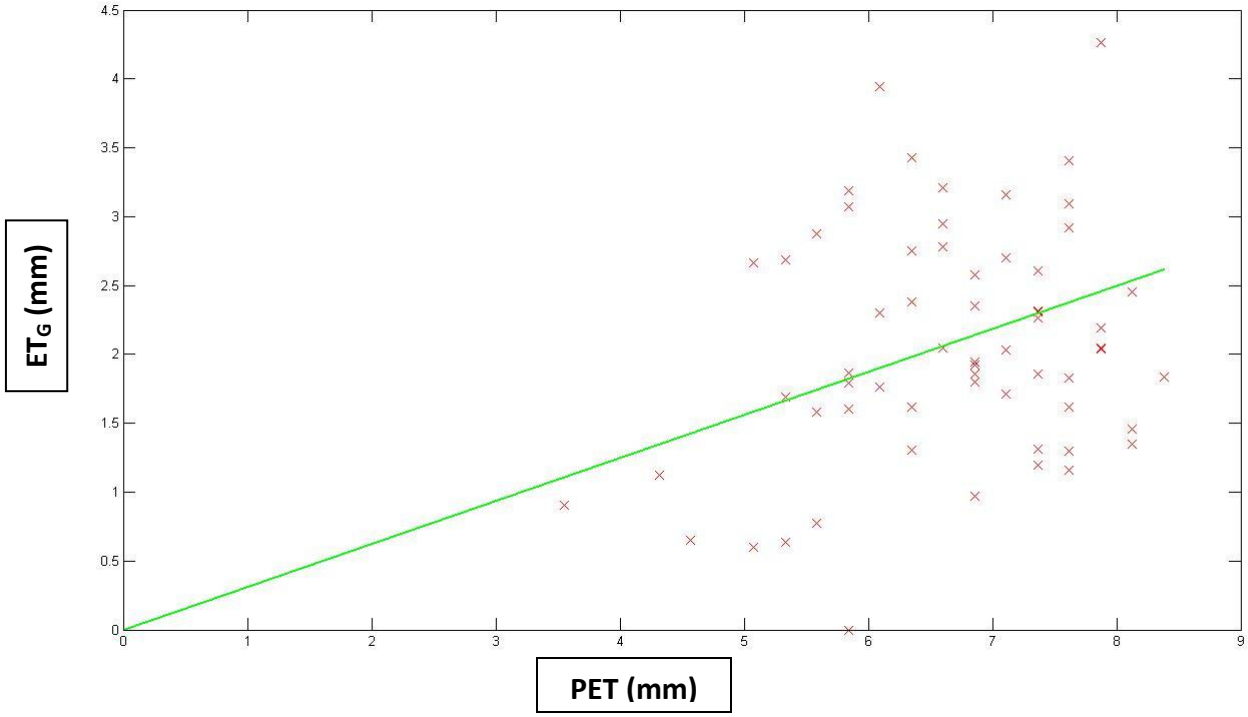

Figure 19: Scatter plot of ET_G against PET with a trend line with a coefficient of 0.31, Oxbow 32

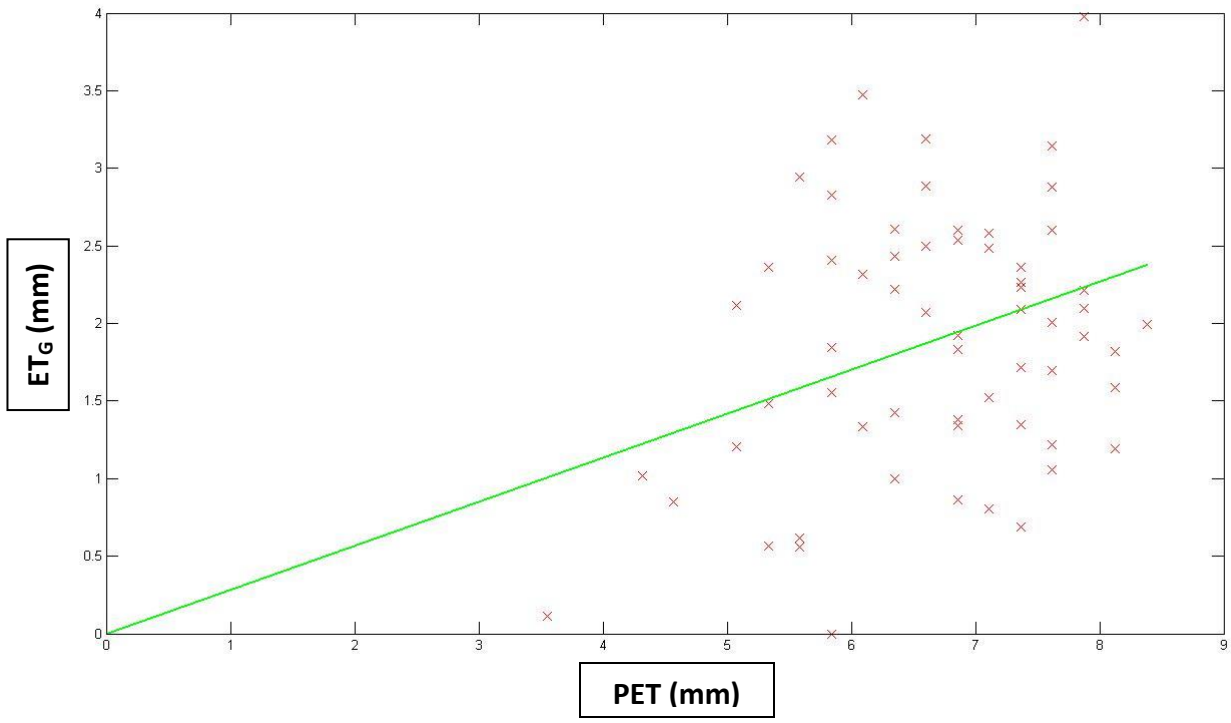

Figure 20: Scatter plot of ET_G against PET with a trend line with a coefficient of 0.28, RPB4

4-Estimation of water loss in soil profile

a) Water retention curve

Here the idea is to estimate the amount of soil water lost during a month by comparing the water retention curves of the beginning and the end of a month. The water retention curve gives the water content of the soil (%) depending on the distance to the water-table (m). Under the water-table the soil is saturated: the water content is at its highest (which depends on the nature of the soil). Above the water-table the soil is unsaturated: the water content decreases following the water retention curve or van Genuchten function [8]. The equation is:

$$\theta(h) = \theta_r + \frac{\theta_s - \theta_r}{(1 + (\alpha h)^n)^{1-\frac{1}{n}}} \qquad (\boldsymbol{Eq\ 5})$$

with θ(%) the water content of the soil, θ_s(cm^3/cm^3) the saturated water content, θ_r(cm^3/cm^3) the residual water content, α (α >0 cm^{-1}) is related to the inverse of the air entry suction, h (cm) the water pressure head or distance to the water-table, and n (n>1) is a measure of the pore-size distribution.

As we saw in previous part, the water-table height drops during summer due to evapotranspiration. As a result the water retention curve changes during this period. Thus, by drawing the water retention curve the first day and the last day of a month we are able to estimate the change in soil water storage which is equivalent to the water loss. Now the question is how to obtain this water retention curve. I used the program Rosetta [9] developed by the United States Department of Agriculture and which estimates unsaturated hydraulic properties from surrogate soil data such as soil texture data or bulk density. Figure 21 provides a view of Rosetta's interface.

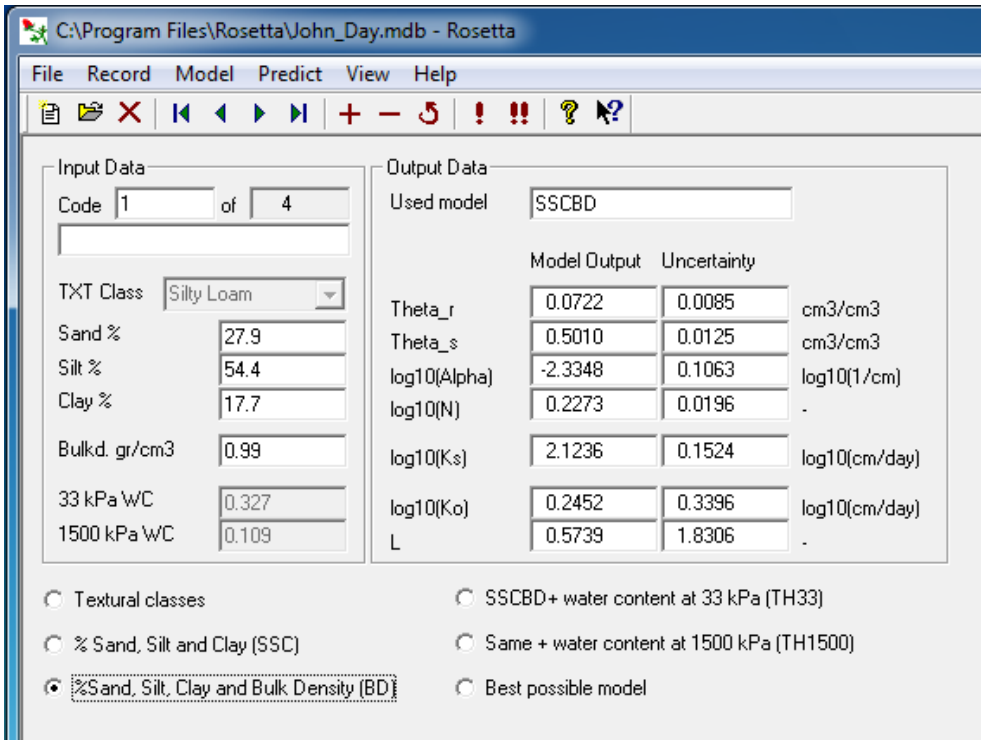

Figure 21: Rosetta's interface

I computed water retention curves for the 1st of July 2010 and the 1st of August 2010 based on Rosetta's output and water-table heights for all the wells. An example is given on figure 20 with Forrest 19. We can clearly see that due to the drop of the water-table the soil water storage decreased during July 2010.

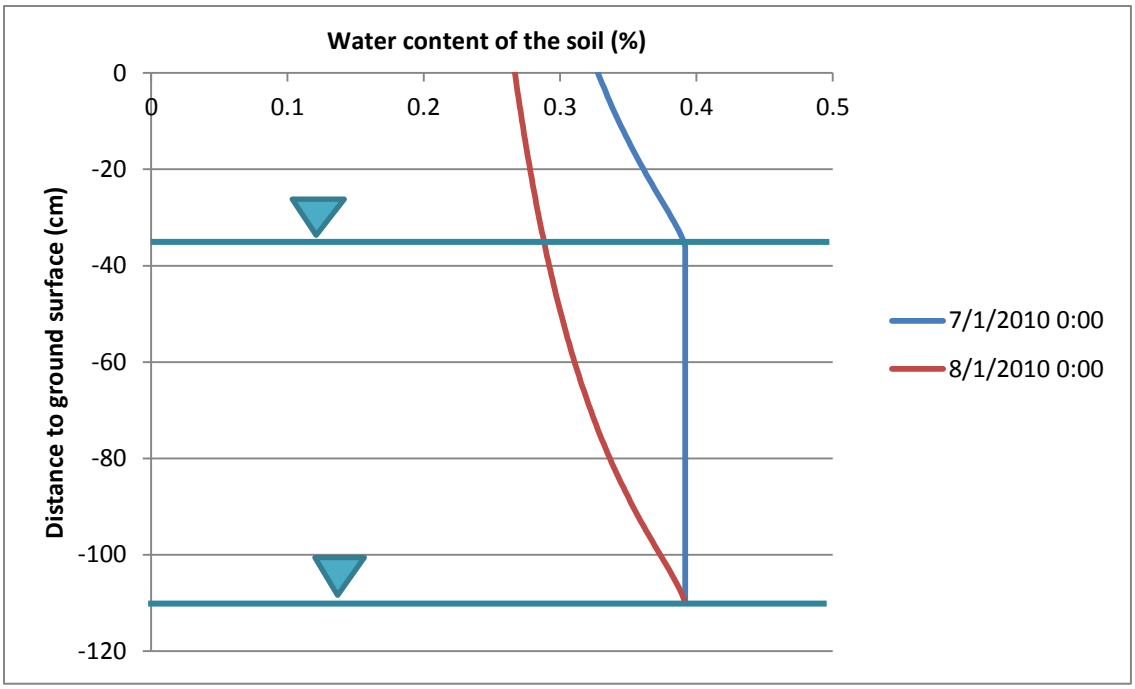

Figure 22: Change in soil water storage during July 2010 at Forrest 19

b) Analysis of the results

To use the software Rosetta I had to find the soil properties that we can see on the left of figure 21. Once again I used the soil survey specific to the MFJD River from the US Department of Agriculture [6]. In fact the soil survey indicates different layers with different soil textures and different thicknesses. This is why I computed the water loss at each well for different soil textures. Table 7 summarizes the well-averaged water losses for different soil textures for July 2010. So we can see that the water loss has the same order of magnitude than the ET_G: the order of the tens of mm (on average, the model predicts a loss of 47mm of water in a month when the ET_G model predicts a loss of 67mm of water in a month). However, there are some discrepancies as depending on the soil texture we chose, the water loss ranges from 30 to 90% of the ET_G. But it is absolutely normal that we have differences: first the software Rosetta cannot represent the exactly the reality, then the way we computed water losses is not perfect, and finally we compare our results to ET_G which is itself found by rough calculations. That is why what really matters is to find the same order of magnitude. Additionally, it is to be said that the ET_G model over predicts the water loss of 10% to 70%. Again, this was expected as the White method usually over estimates the ET_G [2], [3], [5].

Soil Textures	5-20cm layer	20-30cm layer	30-61cm layer	61-99cm layer	Average texture	Well-averaged ET_G (mm/month)
Well-averaged water loss (mm/month)	19	56	61	42	47	67
Standard deviation (mm/month)	8	18	19	13	15	
Water loss / ET_G (%)	29	84	91	63	70	

Table 7: Water loss for different soil textures and ET_G for July 2010

Just to get an idea of the relationship between ET_G and the water loss in the 30-61cm layer case for each well, let us check the scatter plot on figure 23. We can see that we have an important scatter with a standard deviation of 19mm (for a mean of 60mm). But given what we said in previous paragraph it was expected.

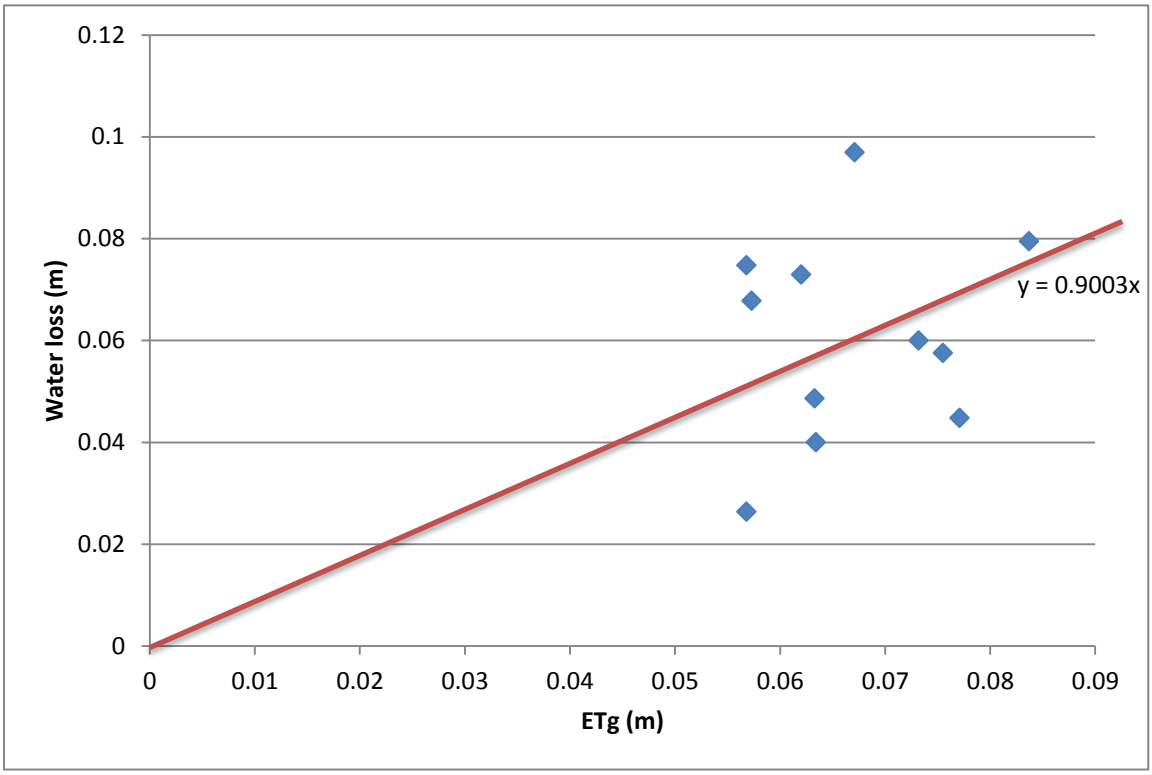

Figure 23: Water loss against ETg for July 2010

III-Analytical approach

1-Presentation of the approach

As we have two sets of wells which are aligned, we are able to plot the shape of the water-table at two different places. Thus, we are able to compare these shapes to the ones that we can obtain with the Boussinesq Equation. The Boussinesq equation is derived from Darcy's Law and describes the water-table in an unconfined, horizontal aquifer draining into a fully penetrating channel (Rupp and Selker, (2005) [10]). We will work under those assumptions even though our system with the MFJD River does not satisfy them rigorously. We will also assume that there is an impermeable bottom boundary where k=0. Everything is summarized on figure 24. By using Boussinesq Equation we will chiefly focus on comparing measured and computed water-tables, and on verifying the value of the saturated hydraulic conductivity k. We used solutions that I computed by myself along with solutions presented in publications ([10], [11]).

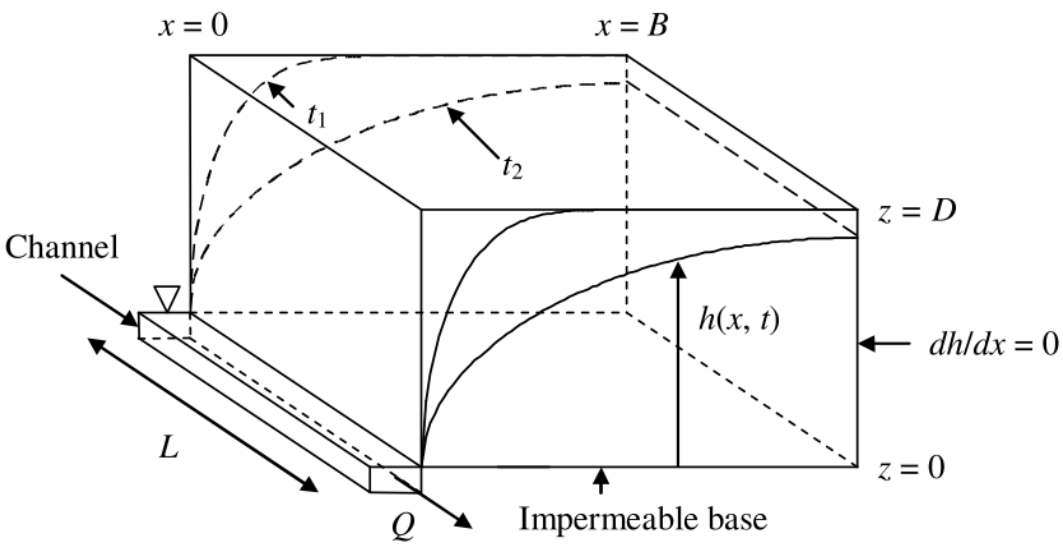

Figure 24: Sketch of the simplified system that we assume to have (from [10])

Boussinesq Equation can be written as follows:

$$\varphi \frac{\partial h}{\partial t} = \frac{\partial}{\partial x}\left(kh\frac{\partial h}{\partial x}\right) + N \qquad (\boldsymbol{Eq\ 6})$$

With φ the drainable porosity, h(m) the water-table elevation, k(m/day) the hydraulic conductivity and N(m/day) a recharge rate which can for instance represent rain or groundwater inflow. Since this Equation is extremely difficult to solve directly, we made different assumptions to solve it in particular cases. We studied first the steady-state case and then solutions that can be qualified as late-time solutions.

2-Steady-state case

a) Description of the case

In this case we assume that the water-table height does not vary in time, so the first derivative of the equation is equal to zero. This assumption appears to be true in the real systems for two periods: at the end of summer when the water-table height is at its lowest and cannot decrease anymore, and at the end of spring when the meadows are fully recharged by rain and snowmelt. The interest of this case is that it turns Boussinesq Equation into a really easy to solve equation and it gives us a first idea of the water-table shape. We computed the steady-state solution of Boussinesq Equation and of the linearized Boussinesq Equation.

b) Boussinesq steady-state

Here we want to solve the steady-state Boussinesq equation with recharge rate which can be written as:

$$\frac{\partial}{\partial x}\left(kh\frac{\partial h}{\partial x}\right) + N = 0 \qquad (\boldsymbol{Eq\ 7})$$

With the boundary condition:

$$h(x=0)=0$$

To solve this equation we used two other equations, Darcy's law and the conservation of mass.

$$u=-k\frac{\partial h}{\partial x}\ with\ u=\frac{q}{h}\xrightarrow{yields} q=-kh\frac{\partial h}{\partial x}\quad Darcy's\ law$$

$$q=-N(B-x)\quad Conservation\ of\ mass \qquad (\boldsymbol{Eq\ 8})$$

With B(m) the right bound of the aquifer (the left being the river). By integration the two expressions of the discharge q we obtain the shape of the water-table height.

$$\boxed{h=\sqrt{\frac{N}{k}(2Bx-x^2)}}$$

The scaled equation being:

$$\boxed{h_+=\sqrt{N_+(2x_+-x_+^2)}}$$

With the scaled variables

$$h_+=\frac{h}{B}\quad x_+=\frac{x}{B}\quad N_+=\frac{N}{k}$$

c) Linearized Boussinesq steady-state

As we will use the linearized equations for the more complicated calculations it is also interesting to solve the linearized steady-state case to check if we obtain a solution comparable to the previous one. The linearized equation can be written as:

$$kh_0\frac{\partial^2 h}{\partial x^2}+N=0 \qquad (\boldsymbol{Eq\ 9})$$

With the boundary conditions

$$h(x=0)=0$$

$$\frac{\partial h}{\partial x}(x=0)=0\ \ (no\ flux\ condition)$$

Integrating twice and using the boundary conditions yields

$$\boxed{h(x)=\frac{N}{2kh_0}(2Bx-x^2)}$$

The scaled form is

$$\boxed{h_+(x_+)=\frac{N_+}{2h_{0+}}(2x_+-x_+^2)}$$

3-Linearized Boussinesq Equation

In this part we solve the Linearized Boussinesq equation without recharge rate. The data suggest that we have long-time processes (several months), which is why we focus on the late-time solution. The linearized Boussinesq Equation is the following one, with the initial and boundary conditions below it:

$$\frac{\partial h}{\partial t} = \frac{kh_0}{\varphi}\frac{\partial^2 h}{\partial x^2} \qquad (\boldsymbol{Eq\ 10})$$

$$\begin{cases} h(x=0,t)=0 \\ \frac{\partial h}{\partial x}(x=B,t)=0 \\ h(x=B,t=0)=D \end{cases}$$

D(m) is the distance between the water-table height at x=B and the bottom boundary where k=0. The late-time solution hypothesis results in the separation of time and space.

$$h(x,t) = X(x)T(t)$$

This leads to the new problem:

$$X\frac{dT}{dt} = \frac{kh_0}{\varphi}T\frac{d^2X}{dx^2} \qquad (\boldsymbol{Eq\ 11})$$

$$\begin{cases} X(x=0)=0 \\ \frac{dX}{dx}(x=B)=0 \\ X(x=B)=D \ and\ T(t=0)=1 \end{cases}$$

Thus we obtain two new equations that we can solve separately.

$$\frac{1}{T}\frac{dT}{dt} = -\lambda \qquad (\boldsymbol{Eq\ 12})$$

$$\frac{kh_0}{\varphi X}\frac{d^2X}{dx^2} = -\lambda \qquad (\boldsymbol{Eq\ 13})$$

With respect to the boundary conditions and the initial condition the final solution is

$$\boxed{h(x,t) = Dsin\left(\frac{\pi}{2B}x\right)e^{-\frac{kh_0\pi^2}{4\varphi B^2}t}}$$

As a result the water-table has the shape of sinus and decreases exponentially with the time. If we assume that h equals to half of its initial value we may compute the time constant that we used in part II-1.

4-Shape of the water-table

a) Figures with the results

In order to see if it is relevant to use Boussinesq solutions in our problem I compared the shape of the water-table obtained with the formula with the real shape measured by the aligned wells at Forrest and Oxbow. As there are only four wells at Forrest and three at Oxbow we cannot have really precise measured shapes. However, these wells are distributed over a 200 meters line which means that the distance is long-enough to give the general shape of the water-table. Figures 25 to 28 show the water-table at Forrest for the first of May, June, July and August of 2010.

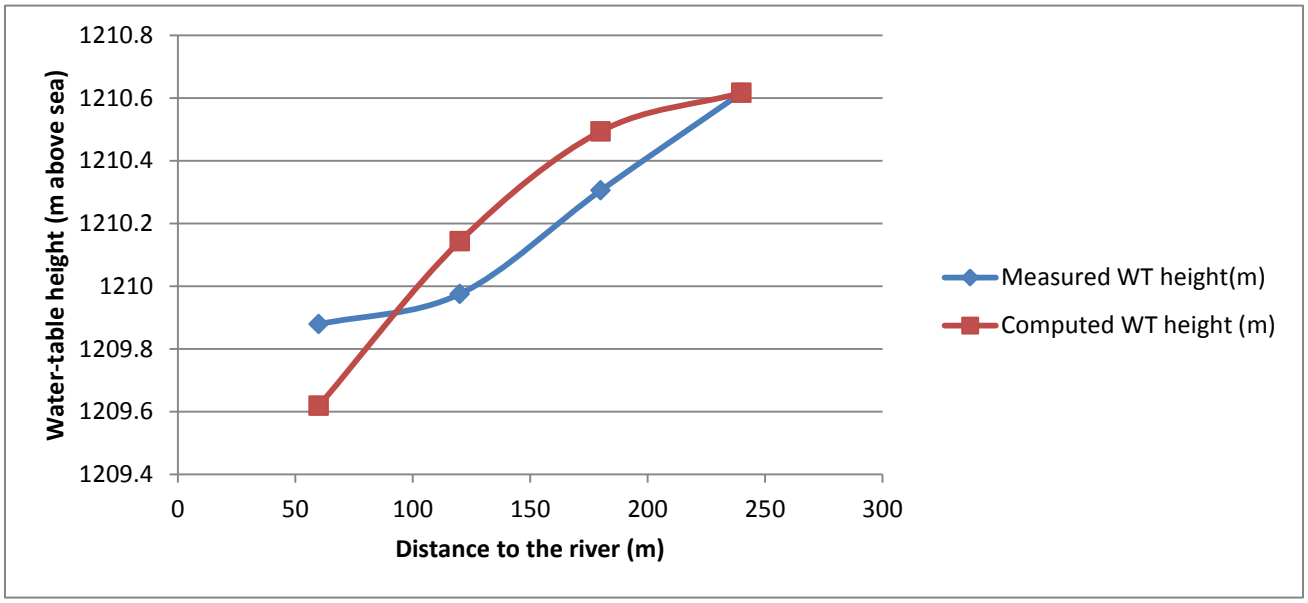

Figure 25: Water-table at Forrest on the 05/01/10

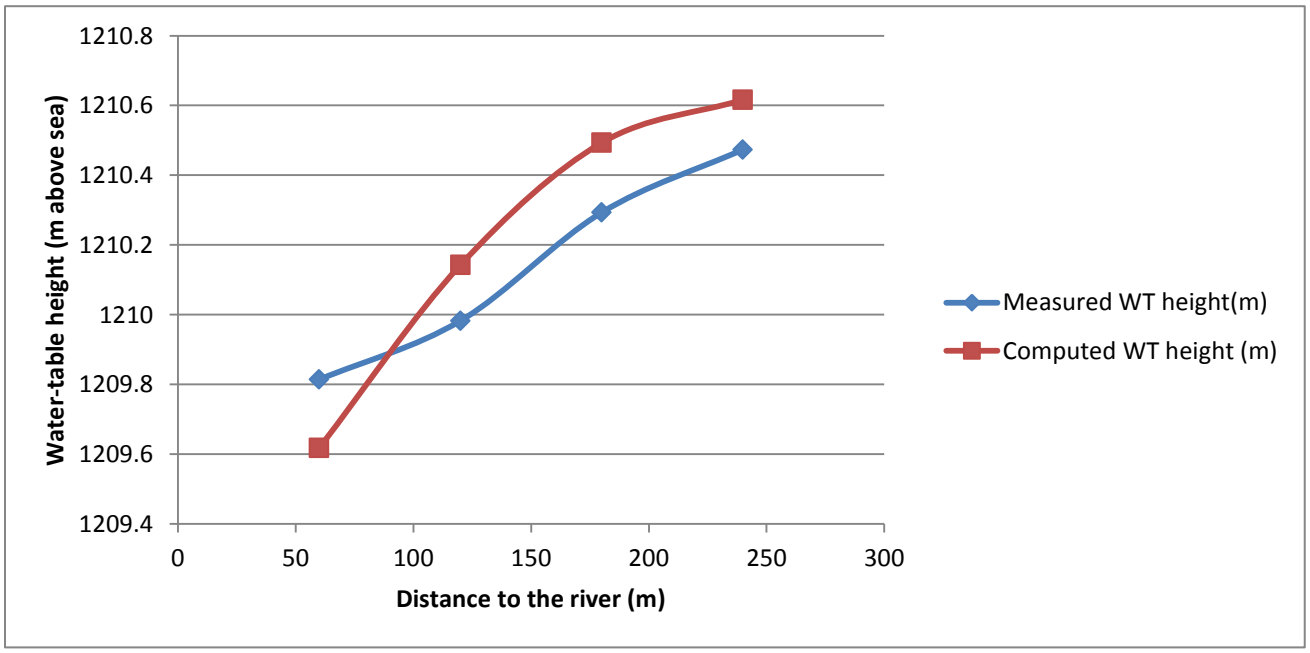

Figure 26: Water-table at Forrest on the 06/01/10

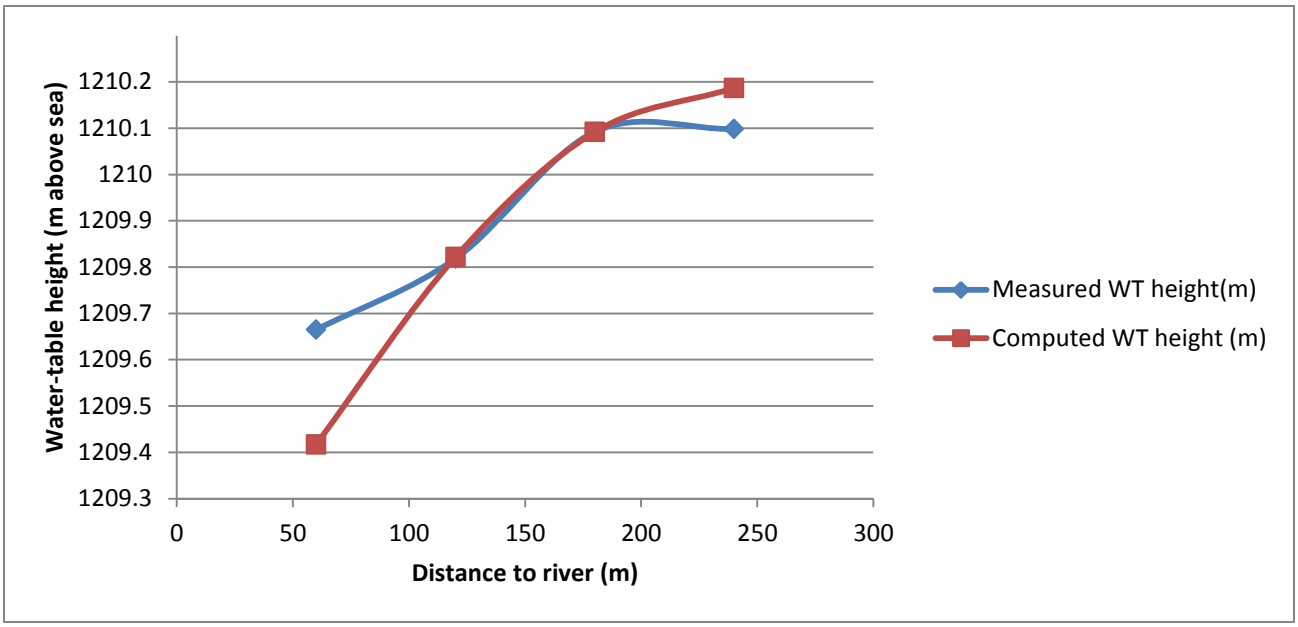

Figure 27: Water-table at Forrest on the 07/01/10

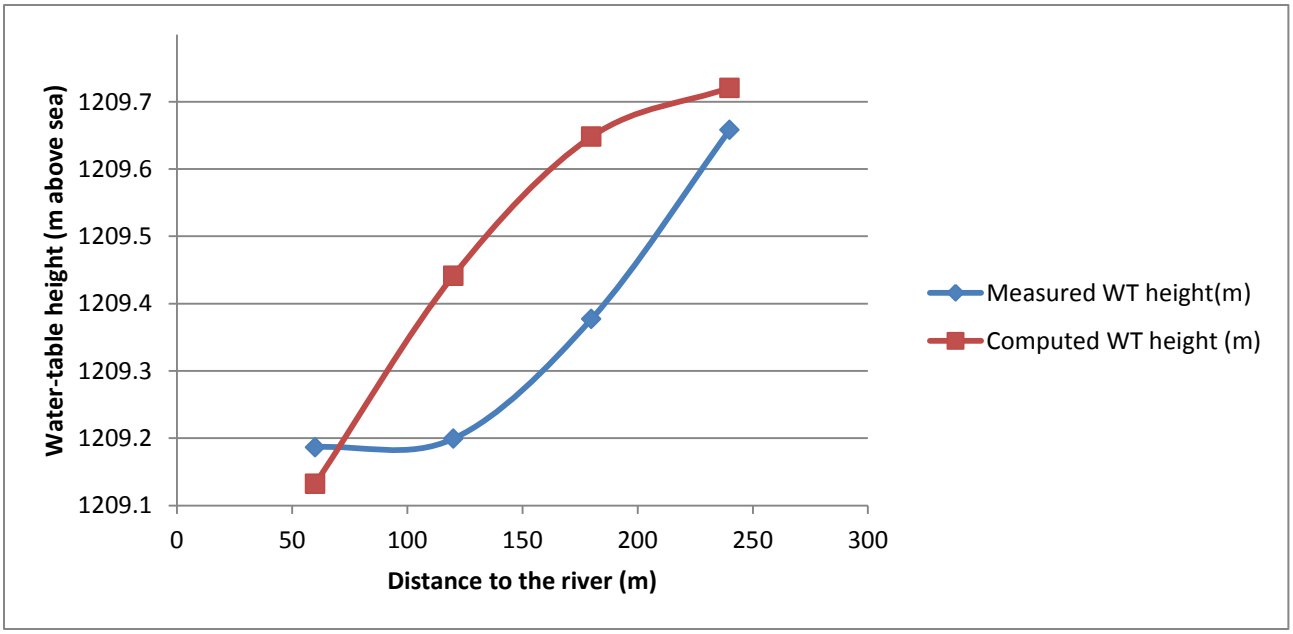

Figure 28: Water-table at Forrest on the 08/01/10

Figures 29 to 32 show the water-table at Oxbow for the same dates.

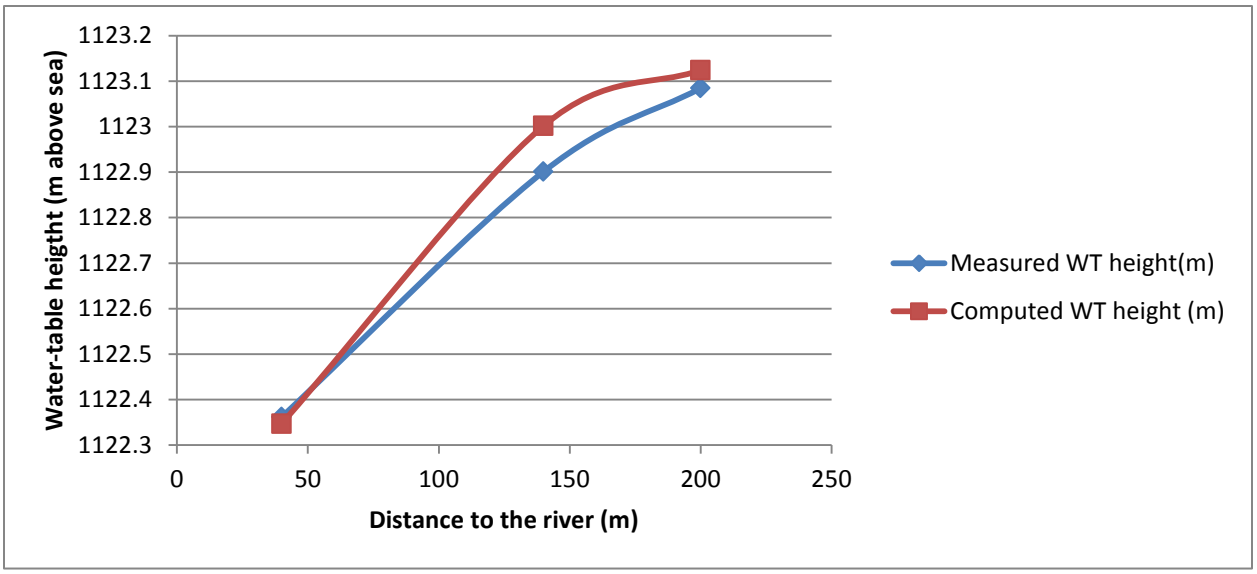

Figure 29: Water-table at Oxbow on the 05/01/10

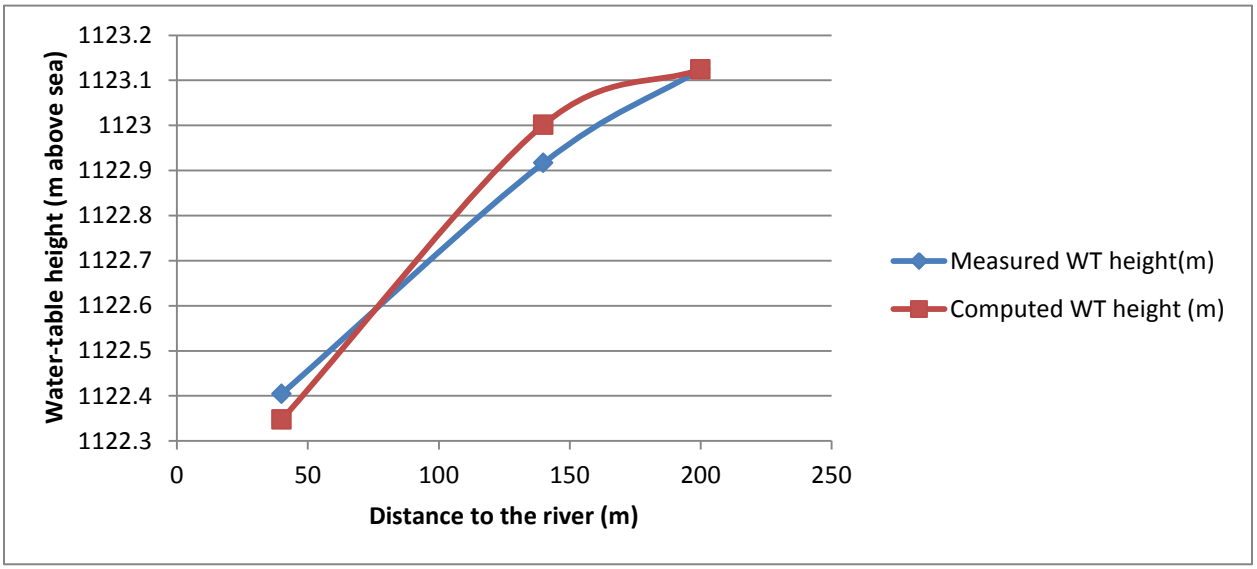

Figure 30: Water-table at Oxbow on the 06/01/10

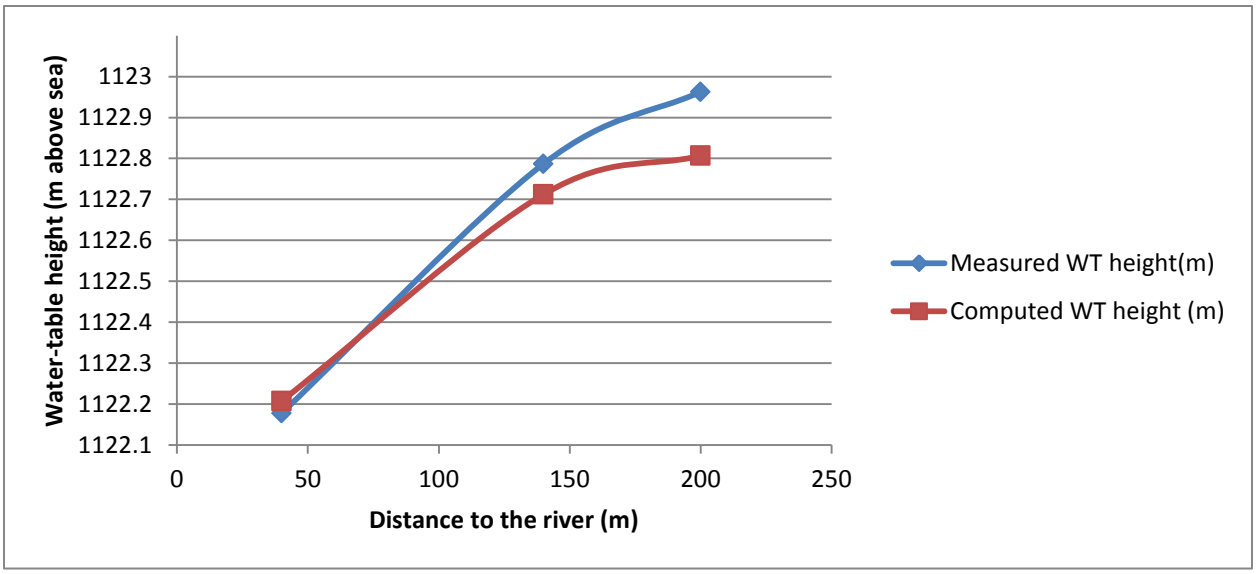

Figure 31: Water-table at Oxbow on the 07/01/10

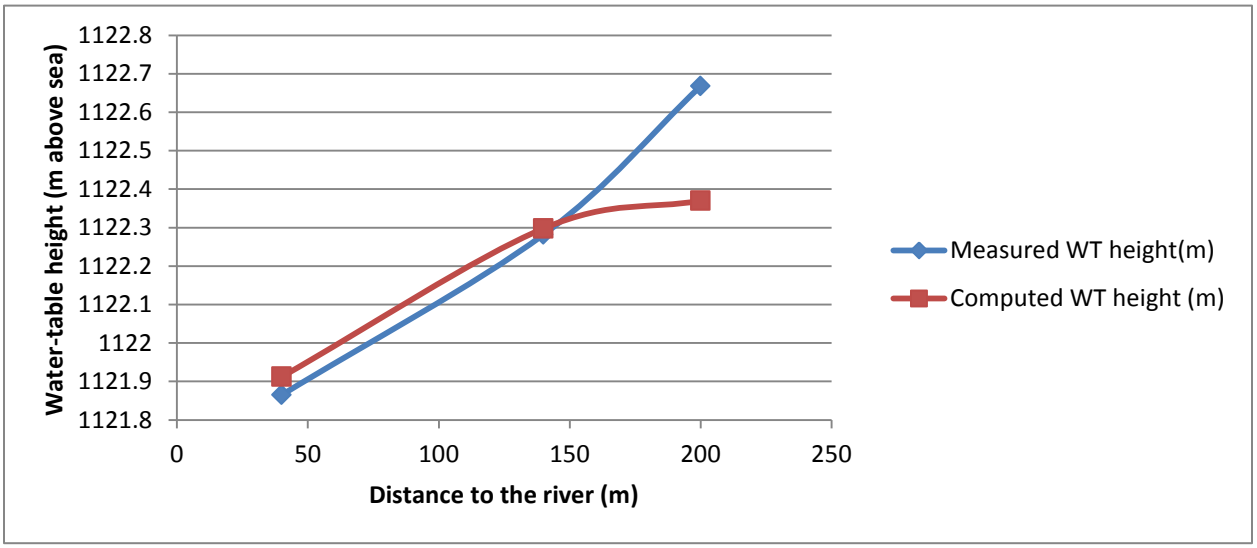

Figure 32: Water-table at Oxbow on the 07/01/10

b) Boussinesq solution and constants

To obtain these curves I used the following constants: k=5m/day (a mean value for the whole problem), B=240m at Forrest and B=200m at Oxbow (position of the last well), φ=0.04 (two times the readily available specific yield), D=1.6m at Forrest and D=1.12m at Oxbow (obtained by choosing the best possible bottom boundary based on the data) and ho=D (chosen arbitrarily). I also used a "recharge rate" which in fact is negative and is set as the mean ET_G subtracted to the mean groundwater upwelling since I could not know when the groundwater upwelling starts but I knew when the ET starts. The goal of computing the solution of the linearized Boussinesq equation was to be able to superimpose such a term (the recharge rate). Indeed Boussinesq only shows the effect of the drainage, and does not show the variations due to ET and groundwater upwelling. To evaluate the effect of those two phenomena I solved the equation:

$$\varphi \frac{dh}{dt} = GW - ET \qquad (\boldsymbol{Eq\ 14})$$

With GW(m/day) the groundwater upwelling, φ the specific yield, h(m) the water-table height and ET(m/day) the evapotranspiration. The solution is:

$$h(t) = \frac{GW - ET}{\varphi} t$$

So the definitive formula for the water-table height is:

$$h(x,t) = Dsin\left(\frac{\pi}{2B}x\right) e^{-\frac{kh_0\pi^2}{4\varphi B^2}t} + \frac{GW - ET}{\varphi} t$$

GW-ET equals -0.0003m/day.

For the time t in days I considered that the decrease of the water-table started on the first of June and that the ET started on the 22nd of June. The values of t I used are presented in table 8.

Date	05/01/10	06/01/10	07/01/10	08/01/10
t for Boussinesq (day)	0	0	30	61
t for the recharge rate (day)	0	0	8	39

Table 8: Values of t for the computation of the water-table height

c) Analysis

First of all, it is to be said that in all the cases the computed water-table height and the measured water-table height are really close to each other. Indeed I computed the difference between the mean measured water-table height and the mean computed water-table height for each month and as we can see with table 9 this difference is always fewer than 10cm. And one should keep in mind that this difference of a few centimeters occurs on distances as wide as 200m.

Date	05/01/10	06/01/10	07/01/10	08/01/10
Difference between measured and computed mean WT at Forrest (cm)	2.4	7.7	3.9	7.0
Difference between measured and computed mean WT at Oxbow (cm)	4.2	0.9	6.7	7.8

Table 9: Difference between measured and computed water-table height

Therefore, we have a good estimation of the position of the water-table when it is at its highest and when it decreases during summer. However, the shape predicted by Boussinesq equation does not perfectly (and sometimes does not at all) fit the "real" shape measured with the wells. This is especially true for the Forrest water-table. As strange as it may seem the measured water-table changes each month and never has

the same shape. And those four measured water-tables never look like the sinus-shaped computed water-tables. But it is difficult to match the theory with the measurements as there were many assumptions made to obtain the linearized Boussinesq solution and there are many things we do not know about the soil: for example a great heterogeneity could explain the strange shapes that we obtain at Forrest. Furthermore, we did not take in heed the slope of the meadows and it could be one of the explanations of the discrepancies given the shape of the measured water-table.

The results are significantly better at Oxbow as first, the evolution of the measured water-table seems relatively coherent, and second, the measured water-tables are not far from the sinus-shaped computed water-tables. Moreover, if instead of assuming that the water-table boundary is situated at the last well we assume that it is situated farther, 50m farther for example (so B=250m instead of 200m), the results are even better as we can see on figures 33 and 34. And this new assumption is not even contradicting anything we used to solve the problem. I fact, figures 33 and 34 would suggest that the small difference between the two curves is only due to the slope that we did not take into account in the model.

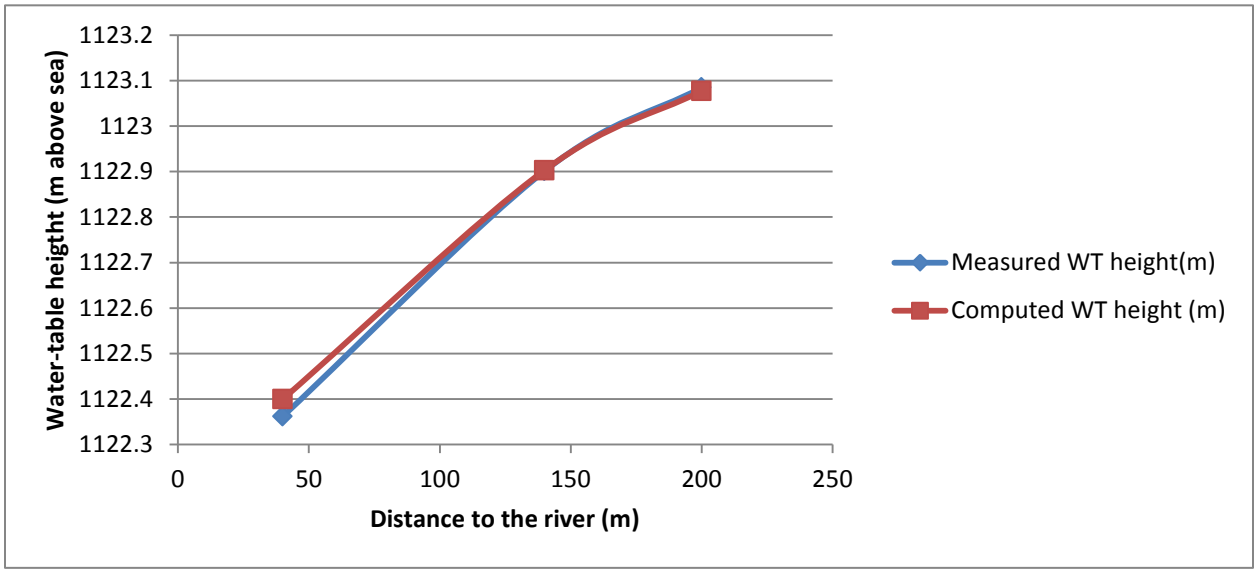

Figure 33: Water-table at Oxbow on the 05/01/10 with modified B

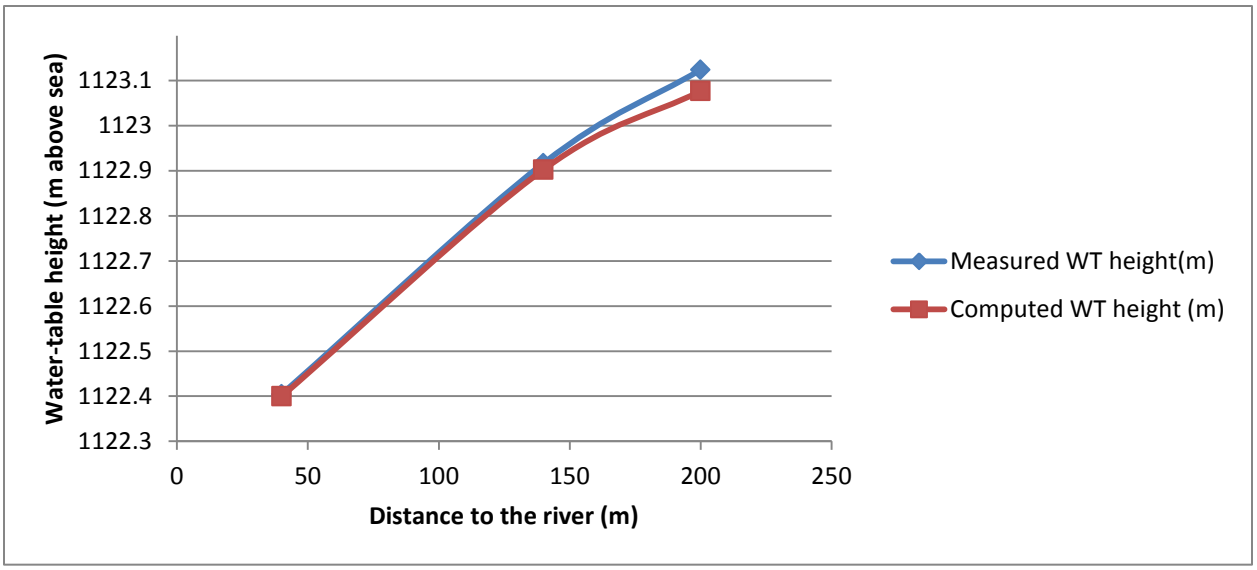

Figure 34: Water-table at Oxbow on the 06/01/10 with modified B

To solve the linearized Boussinesq equation we assumed that we were looking for a "late-time" solution which led us to the separation of time and space. This is why the computed water-table has always the same shape and just drops with the time. But the data suggest that the shape is actually changing with the time, going from a convex sinus shape to a concave shape. This is something else that we did not take in heed and that also led to the differences between measured and computed shapes.

What is important to keep in mind is that the results I am presenting here are not extremely precise. I made a lot of assumptions and I had to choose arbitrarily some constants, as the elevation of the bottom boundary which has a non-insignificant influence on the results.

5-Verification of the hydraulic conductivity's value

As we saw before, the hydraulic conductivity ranges from 0.02 to 20 m/day according to the soil survey of the Grant County [7]. But we said that a good mean value was 5m/day. Here we will compare all the values of k that we used or that we computed a check if the theory matches the fields measurements.

Confirmation of the values from the soil survey with Rosetta

I already mentioned the fact that I used two different soil surveys, on that was done for the whole Grant County [7] and one that I find more accurate and which was done near the MFJD River [6]. However, the second one does not provide directly the hydraulic conductivity. Fortunately, the software Rosetta computes the terms of the Van Genuchten-Mualem model which allow us to compute k [9]. The equation is the following one:

$$k = k_0 S_e^L \left(1 - \left(1 - S_e^{\frac{n}{n-1}}\right)^{1-\frac{1}{n}}\right)^2 \qquad (\boldsymbol{Eq\ 15})$$

with k_0 the matching point at saturation (cm/day), L an empirical pore tortuosity/connectivity parameter, n the same curve shape parameter than in the water retention curve and S_e the relative saturation which expression is:

$$S_e = \frac{\theta(h) - \theta_r}{\theta_s - \theta_r}$$

When computing k with this formula, I find that k increases with the depth. It ranges from 0.2m/day at the surface to 17m/day deep inside the soil. In summary, it seems that the two soil surveys agree about the values of the hydraulic conductivity.

Confirmation of the mean value by the computed water-table

We have just seen that our model represents quite well the drop of the water-table during summer. Indeed, at Forrest, even if the computed shape is not the same than the real shape the solution of Boussinesq equation falls at the same speed than the real water-table. And at Oxbow it is also true and in addition the computed shape matches well the real one. To be able to have a model that has a good temporal evolution I had to set an optimal value of the hydraulic conductivity. And after several tries I found that the best value was k=5m/day. It is rather good as it is a mean value or so regarding the soil surveys' values.

Analytical method to determine the hydraulic conductivity

To make a more rigorous verification of the value of the hydraulic conductivity, I used an analytical method that I found in the publication: "Analytical methods for estimating saturated hydraulic conductivity in a tile-drained field" by Rupp, Owens, Warren and Selker [11]. The idea is simple: they start from an exact solution of Boussinesq equation without recharge rate which is:

$$h(x,t) = \frac{D\phi\left(\frac{x}{B}\right)}{1 + 1.15\left(\frac{kD}{\varphi B^2}\right)t} \qquad (\boldsymbol{Eq\ 16})$$

where ϕ(x/B) is the initial form of the free surface and is described by an inverse incomplete Beta function.

Then they assume that D=h(B,0) and they take the water-table height at two different times to obtain this expression for k:

$$k = \frac{\varphi B^2}{1.115h(B,t_1)}\left[\frac{h(x,t_1)}{h(x,t_2)} - 1\right]\frac{1}{(t_2 - t_1)}$$

I computed k for the two water-tables at Forrest and Oxbow. I used the same values of φ and B than on previous part. I took the 06/01/10 for t_1 and the 08/01/10 for t_2. As here the formula deals with a system where only drainage occurs (h(x,t) is a solution of the Boussinesq equation without recharge rate) I had to

correct the value of $h(x,t_2)$ in order to be sure that the variation of the water-table height was only due to drainage into the river. I did exactly like in previous part except that instead of adding the negative recharge rate representing groundwater inflow minus ET, I subtracted it. Like in previous part I set the beginning of the ET 39 days prior to the 08/01/10. The values that I found for the hydraulic conductivity are presented in table 10.

k Forrest (m/day)	k Oxbow (m/day)
5.87	5.45

First I was surprised to find values so close to the mean value I found before. I was expecting something between 1 and 20m/day but it seems that 5m/day is an accurate average value for the hydraulic conductivity along the MFJD River, although we have again to be careful and understand that those results would change if the values of some parameters were to be changed.

Conclusion

The main goals of this project were first to characterize the system composed by the Middle Fork John Day River and the water-table of the adjoining meadows and secondly to estimate the different fluxes between the river, the water-table the surrounding environment. The system was characterized by several constants among which the hydraulic conductivity evaluated at 5m/day, the specific yield evaluated at 0.04, the readily available specific yield evaluated at 0.02 and also the water content of the soil evaluated at 10 to 45%. However, the soil surveys indicate that those values vary a lot with the distance to the ground surface because of the heterogeneity of the soil and the presence of several layers. The water-table was characterized by its shape even if we failed to find an analytical model that describes this shape in all circumstances. Our model uses a sinus-shaped curve but had consistent results only with the field measurements from Oxbow. We also estimated the main fluxes that occur in the system: a groundwater evapotranspiration of 2.2mm/day that occurs only during summer, a groundwater upwelling of 1.9mm/day and drainage of soil water to the stream that ranges between 0.002 to 0.02cfs added per kilometer of river and that has a time constant of 100 days. These results about the drainage suggest that this phenomenon is useful to the river as it prevents the river to dry up completely at the end of the summer. We confirmed our estimation of the evapotranspiration by evaluating the amount of water lost in soil profile: both methods gave agreeing values of the drop of the water-table (e.g. about 70cm during July at Forrest 19). A longer and more profound study would give better estimations as we did many assumptions and rough estimations. In particular, using more data (several years and more wells) could confirm the results. It would also be interesting to do a numerical analysis of the system, with Hydrus 1D for instance, to check the analytical and experimental values.

Bibliography

[1] Upper Middle Fork John Day River, Intensively Monitored Watershed: Experimental Design and Implementation Plan, Upper Middle Fork John Day Workin Group, (2010).
http://www.middleforkimw.org/MFIMW_Design&ImplementationPlan.pdf

[2] Estimating groundwater evapotranspiration rates using diurnal water-table fluctuations in a semi-arid riparian zone, Laura K. Lautz, (2007)

[3] Estimation of groundwater consumption by phreatophytes using diurnal water-table fluctuations: A saturated-unsaturated flow assessment, Steven P. Loheide II, James J. Butler Jr., Steven M. Gorelick, (2005)
https://eng.ucmerced.edu/people/rbales/Courses/ES214files/2005WR003942

[4] Agrimet Website
http://www.usbr.gov/pn/agrimet/wxdata.html

[5] A method of estimating ground-water supplies based on discharge by plants and evaporation from soil: results of investigations in Escalante Valley, Utah, With WN, 51932), US Geol Surv Water Suppl Pap 659-A

[6] Soil Survey near the MFJD River
http://ssldata.nrcs.usda.gov/querypage.asp?chksa=1&ac=244&as=3991#sitevar

[7] Soil Survey of the Grant County
http://soils.usda.gov/survey/online_surveys/oregon/OR633/Grant.pdf

[8] A closed-form equation for predicting the hydraulic conductivity of unsaturated soils, M. van Genuchten, (1980), Soil Sci. Soc. Am. J., 44, 892 – 898

[9] Software Rosetta with documentation
http://www.ars.usda.gov/Services/docs.htm?docid=8953

[10] Drainage of a horizontal Boussinesq aquifer with a power law hydraulic conductivity profile,
David E. Rupp and John S. Selker, (2005)

[11] Analytical methods for estimating saturated hydraulic conductivity in a tile-drained field
David E. Rupp, Joshua M. Owens, Kristina L. Warren1, John S. Selker, (2003)